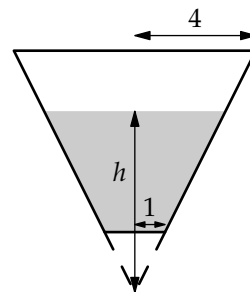


We see that the filter is a **frustum**, which is a cone with a smaller cone chopped off. The volume of a frustum is most easily computed as the difference in volume between the two cones. To compute the volumes of the cones, we need their heights. So we will be much better off if our height is measured from the vertex of the imaginary cones, not from the bottom of the filter.



Next, the formula for the volume of a cone is $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height. So we need to determine how the height is related to the radius. They are directly proportional, as we can see from our picture at right. In particular, decreasing the height by 6 causes a decrease of 3 in the radius, so we conclude that $h/r = 2$, and thus the radius at height h is $h/2$. Note that the top of our frustum is at $h = 8$ (not $h = 6$), and at the top we have radius $8/2 = 4$, as expected. The bottom of the frustum is at $h = 2 \cdot 1 = 2$.

So the volume of the water at height h is:

$$V = \frac{1}{3}\pi r^2 h - \frac{1}{3}\pi(1)^2(2) = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h - \frac{1}{3}\pi(1)^2(2) = \frac{1}{12}\pi h^3 - \frac{2}{3}\pi.$$

We can differentiate this equation to get an equation relating the rates:

$$V' = \frac{1}{4}\pi h^2 h'.$$

We are told that $V' = -2$, as the rate of decrease of volume is constant. But we also need to find the value of h in order to solve for h' . So we need to find the volume at $t = 30$ and compute its height.

We can start by finding the initial volume (at $t = 0$):

$$V(0) = \frac{1}{12}\pi(8)^3 - \frac{2}{3}\pi.$$

This simplifies to 42π .

When $t = 30$, we will have lost $2 \cdot 30 = 60 \text{ cm}^3$ of water, so the volume will be $V(30) = 42\pi - 60$. We can plug this in to our equation for volume to find h at time $t = 30$:

$$42\pi - 60 = \frac{1}{12}\pi h^3 - \frac{2}{3}\pi.$$

This gives $h^3 = \frac{512\pi - 720}{\pi}$, and thus $h = \sqrt[3]{\frac{512\pi - 720}{\pi}} \approx 6.564$.

Finally, we can go back to our related rates equation:

$$-2 = \frac{1}{4}\pi h^2 h',$$

and plug in the found value of h . This gives:

$$-2 = \frac{1}{4}\pi\left(\frac{512\pi - 720}{\pi}\right)^{\frac{2}{3}} h'$$

We solve for h' to get our answer:

$$h' = -\frac{8}{\sqrt[3]{\pi(512\pi - 720)^{\frac{2}{3}}}} = -\frac{2}{\sqrt[3]{\pi(64\pi - 90)^{\frac{2}{3}}}}.$$

This is approximately -0.0591 , and thus at time $t = 30$ the water level is decreasing at a rate of 0.0591 cm/sec . \square