AMC 12/AHSME 1961

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1 When simplified, $(-\frac{1}{125})^{-2/3}$ becomes:

(A) $\frac{1}{25}$ **(B)** $-\frac{1}{25}$ **(C)** 25 **(D)** -25 **(E)** $25\sqrt{-1}$

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2 An automobile travels $a/6$ feet in r seconds. If this rate is maintained for 3 minutes, how many yards does it travel in 3 minutes?

(A) $\frac{a}{1080r}$ **(B)** $\frac{30r}{a}$ **(C)** $\frac{30a}{r}$ **(D)** $\frac{10r}{a}$ **(E)** $\frac{10a}{r}$

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3 If the graphs of $2y+x+3=0$ and $3y+ax+2=0$ are to meet at right angles, the value of a is:

(A) $\pm\frac{2}{3}$ **(B)** $-\frac{2}{3}$ **(C)** $-\frac{3}{2}$ **(D)** 6 **(E)** -6

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4 Let the set consisting of the squares of the positive integers be called u; thus u is the set $1, 4, 9, 16....$ If a certain operation on one or more members of the set always yields a member of the set, we say that the set is closed under that operation. Then u is closed under:

(A) Addition **(B)** Multiplication **(C)** Division **(D)** Extraction of a positive integral root **(E)**Non

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5 Let $S=(x-1)^4+4(x-1)^3+6(x-1)^2+4(x-1)+1$. Then S equals:

(A) $(x-2)^4$ **(B)** $(x-1)^4$ **(C)** x^4 **(D)** $(x+1)^4$ **(E)** x^4+1

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6 When simplified, $\log 8 \div \log \frac{1}{8}$ becomes:

(A) $6\log 2$ **(B)** $\log 2$ **(C)** 1 **(D)** 0 **(E)** -1

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7 When simplified, the third term in the expansion of $\left(\frac{a}{\sqrt{x}}-\frac{\sqrt{x}}{a^2}\right)^6$ is:

(A) $\frac{15}{x}$ **(B)** $-\frac{15}{x}$ **(C)** $-\frac{6x^2}{a^9}$ **(D)** $\frac{20}{a^3}$ **(E)** $-\frac{20}{a^3}$

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8 Let the two base angles of a triangle be A and B, with B larger than A. The altitude to the base divides the vertex angle C into two parts, C_1 and C_2, with C_2 adjacent to side a. Then:

(A) $C_1+C_2=A+B$ **(B)** $C_1-C_2=B-A$ **(C)** $C_1-C_2=A-B$ **(D)** $C_1+C_2=B-A$ **(E)**

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9 Let r be the result of doubling both the base and exponent of a^b, $b \neq 0$. If r equals the product of a^b by x^b, then x equals:

(A)a **(B)** $2a$ **(C)** $4a$ **(D)** 2 **(E)** 4

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10 Each side of triangle ABC is 12 units. D is the foot of the perpendicular dropped from A on BC, and E is the midpoint of AD. The length of BE, in the same unit, is:

(A) $\sqrt{18}$ **(B)** $\sqrt{28}$ **(C)** 6 **(D)** $\sqrt{63}$ **(E)** $\sqrt{98}$

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11 Two tangents are drawn to a circle from an exterior point A; they touch the circle at points B and C respectively. A third tangent intersects segment AB in P and AC in R, and touches the circle at Q. If $AB = 20$, then the perimeter of triangle APR is

(A) 42 **(B)** 40.5 **(C)** 40 **(D)** $39\frac{7}{8}$ **(E)** not determined by the given information

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12 The first three terms of a geometric progression are $\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[6]{2}$. Find the fourth term.

(A) 1 **(B)** $\sqrt[7]{2}$ **(C)** $\sqrt[8]{2}$ **(D)** $\sqrt[9]{2}$ **(E)** $\sqrt[10]{2}$

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13 The symbol $|a|$ means a is a positive number or zero, and $-a$ if a is a negative number. For all real values of t the expression $\sqrt{t^4 + t^2}$ is equal to:

(A) t^3 **(B)** $t^2 + t$ **(C)** $|t^2 + t|$ **(D)** $t\sqrt{t^2 + 1}$ **(E)** $|t|\sqrt{1 + t^2}$

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14 A rhombus is given with one diagonal twice the length of the other diagonal. Express the side of the rhombus is terms of K, where K is the area of the rhombus in square inches.

(A) $\sqrt{K}$ **(B)** $\frac{1}{2}\sqrt{2K}$ **(C)** $\frac{1}{3}\sqrt{3K}$ **(D)** $\frac{1}{4}\sqrt{4K}$ **(E)** None of these are correct

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15 If x men working x hours a day for x days produce x articles, then the number of articles (not necessarily an integer) produced by y men working y hours a day for y days is:

(A) $\frac{x^3}{y^2}$ **(B)** $\frac{y^3}{x^2}$ **(C)** $\frac{x^2}{y^3}$ **(D)** $\frac{y^2}{x^3}$ **(E)** y

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16 An altitude h of a triangle is increased by a length m. How much must be taken from the corresponding base b so that the area of the new triangle is one-half that of the original triangle?

(A) $\frac{bm}{h+m}$ **(B)** $\frac{bh}{2h+2m}$ **(C)** $\frac{b(2m+h)}{m+h}$ **(D)** $\frac{b(m+h)}{2m+h}$ **(E)** $\frac{b(2m+h)}{2(h+m)}$

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17 In the base ten number system the number 526 means $5 \cdot 10^2 + 2 \cdot 10 + 6$. In the Land of Mathesis, however, numbers are written in the base r. Jones purchases an automobile there for 440 monetary units (abbreviated m.u). He gives the salesman a 1000 m.u bill, and receives, in change, 340 m.u. The base r is:

(A) 2 **(B)** 5 **(C)** 7 **(D)** 8 **(E)** 12

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18 The yearly changes in the population census of a town for four consecutive years are, respectively, 25% increase, 25% increase, 25% decrease, 25% decrease. The net change over the four years, to the nearest percent, is:

(A) -12 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 12

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19 Consider the graphs of $y = 2\log x$ and $y = \log 2x$. We may say that:

(A) They do not intersect **(B)** They intersect at 1 point only **(C)** They intersect at 2 points only
(D) They intersect at a finite number of points but greater than 2 **(E)** They coincide

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20 The set of points satisfying the pair of inequalities $y > 2x$ and $y > 4 - x$ is contained entirely in quadrants:

(A) I and II **(B)** II and III **(C)** I and III **(D)** III and IV **(E)** I and IV

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21 Medians AD and and CE of triangle ABC intersect in M. The midpoint of AE is N. Let the area of triangle MNE be k times the area of triangle ABC. Then k equals:

(A) $\frac{1}{6}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{16}$

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22 If $3x^3 - 9x^2 + kx - 12$ is divisible by $x - 3$, then it is also divisible by:

(A) $3x^2 - x + 4$ **(B)** $3x^2 - 4$ **(C)** $3x^2 + 4$ **(D)** $3x - 4$ **(E)** $3x + 4$

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23 Points P and Q are both in the line segment AB and on the same side of its midpoint. P divides AB in the ratio $2:3$, and Q divides AB in the ratio $3:4$. If $PQ = 2$, then the length of AB is:

(A) 60 **(B)** 70 **(C)** 75 **(D)** 80 **(E)** 85

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24 Thirty-one books are arranged from left to right in order of increasing prices. The price of each book differs by $\$2$ from that of each adjacent book. For the price of the book at the extreme right a customer can buy the middle book and the adjacent one. Then:

(A) The adjacent book referred to is at the left of the middle book **(B)** The middle book sells for \$36
(C) The cheapest book sells for \$4 **(D)** The most expensive book sells for \$64 **(E)** None of these

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25 Triangle ABC is isosceles with base AC. Points P and Q are respectively in CB and AB and such that $AC = AP = PQ = QB$. The number of degrees in angle B is:

(A) $25\frac{5}{7}$ **(B)** $26\frac{1}{3}$ **(C)** 30 **(D)** 40 **(E)** Not determined by the information given

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26 For a given arithmetic series the sum of the first 50 terms is 200, and the sum of the next 50 terms is 2700. The first term in the series is:

(A) -1221 **(B)** -21.5 **(C)** -20.5 **(D)** 3 **(E)** 3.5

27 Given two equiangular polygons P_1 and P_2 with different numbers of sides; each angle of P_1 is x degrees and each angle of P_2 is kx degrees, where k is an integer greater than 1. The number of possibilities for the pair (x, k) is:

(A) infinite **(B)** finite, but greater than 2 **(C)** Two **(D)** One **(E)** Zero

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28 If 2137^{753} is multiplied out, the units' digit in the final product in the final product is:

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

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29 Let the roots of $ax^2 + bx + c = 0$ be r and s. The equation with roots $ar + b$ and $as + b$ is:

(A) $x^2 - bx - ac = 0$ **(B)** $x^2 - bx + ac = 0$ **(C)** $x^2 + 3bx + ca + 2b^2 = 0$ **(D)** $x^2 + 3bx - ca + 2b^2 = 0$
(E) $x^2 + bx(2 - a) + a^2c + b^2(a + 1) = 0$

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30 If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then $\log_5 12 =$?

(A) $\frac{a+b}{a+1}$ **(B)** $\frac{2a+b}{a+1}$ **(C)** $\frac{a+2b}{1+a}$ **(D)** $\frac{2a+b}{1-a}$ **(E)** $\frac{a+2b}{1-a}$

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31 In triangle ABC the ratio $AC : CB$ is $3 : 4$. The bisector of the exterior angle at C intersects BA extended at P (A is between P and B). The ratio $PA : AB$ is:

(A) $1 : 3$ **(B)** $3 : 4$ **(C)** $4 : 3$ **(D)** $3 : 1$ **(E)** $7 : 1$

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32 A regular polygon of n sides is inscribed in a circle of radius R. The area of the polygon is $3R^2$. Then n equals:

(A) 8 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 18

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33 The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** More than three, but finite

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34 Let S be the set of values assumed by the fraction

$$\frac{2x+3}{x+2}$$

when x is any member of the interval $x \geq 0$. If there exists a number M such that no number of the set S is greater than M, then M is an upper bound of S. If there exists a number m such that such that no number of the set S is less than m, then m is a lower bound of S. We may then say:

(A) m is in S, but M is not in S **(B)** M is in S, but m is not in S **(C)** Both m and M are in S **(D)** Neither m nor M
(E) M does not exist either in or outside S

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35 The number 695 is to be written with a factorial base of numeration, that is, $695 = a_1 + a_2 \times 2! + a_3 \times 3! + ...a_n \times n!$ where $a_1, a_2, a_3...a_n$ are integers such that $0 \le a_k \le k$, and $n!$ means $n(n-1)(n-2)...2 \times 1$. Find a_4

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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36 In triangle ABC the median from A is given perpendicular to the median from B. If $BC = 7$ and $AC = 6$, find the length of AB.

(A) 4 **(B)** $\sqrt{17}$ **(C)** 4.25 **(D)** $2\sqrt{5}$ **(E)** 4.5

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37 In racing over a distance d at uniform speed, A can beat B by 20 yards, B can beat C by 10 yards, and A can beat C by 28 yards. Then d, in yards, equals:

(A) Not determined by the given information **(B)** 58 **(C)** 100 **(D)** 116 **(E)** 120

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38 Triangle ABC is inscribed in a semicircle of radius r so that its base AB coincides with diameter AB. Point C does not coincide with either A or B. Let $s = AC + BC$. Then, for all permissible positions of C:

(A) $s^2 \le 8r^2$ **(B)** $s^2 = 8r^2$ **(C)** $s^2 \ge 8r^2$ **(D)** $s^2 \le 4r^2$ **(E)** $x^2 = 4r^2$

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39 Any five points are taken inside or on a square with side length 1. Let a be the *smallest* possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is equal to or less than a. Then a is:

(A) $\sqrt{3}/3$ **(B)** $\sqrt{2}/2$ **(C)** $2\sqrt{2}/3$ **(D)** 1 **(E)** $\sqrt{2}$

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40 Find the minimum value of $\sqrt{x^2 + y^2}$ if $5x + 12y = 60$.

(A) $\frac{60}{13}$ **(B)** $\frac{13}{5}$ **(C)** $\frac{13}{12}$ **(D)** 1 **(E)** 0
