AMC 12/AHSME 1965

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by AIME15, rrusczyk

1 The number of real values of x satisfying the equation $2^{2x^2-7x+5}=1$ is:
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 4

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2 A regular hexagon is inscribed in a circle. The ratio of the length of a side of the hexagon to the length of the shorter of the arcs intercepted by the side, is:
(A) $1:1$ **(B)** $1:6$ **(C)** $1:\pi$ **(D)** $3:\pi$ **(E)** $6:\pi$

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3 The expression $(81)^{-2^{-2}}$ has the same value as:
(A) $\frac{1}{81}$ **(B)** $\frac{1}{3}$ **(C)** 3 **(D)** 81 **(E)** 81^4

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4 Line l_2 intersects line l_1 and line l_3 is parallel to l_1. The three lines are distinct and lie in a plane. The number of points equidistant from all three lines is:
(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

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5 When the repeating decimal $0.363636\ldots$ is written in simplest fractional form, the sum of the numerator and denominator is:
(A) 15 **(B)** 45 **(C)** 114 **(D)** 135 **(E)** 150

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6 If $10^{\log_{10}9}=8x+5$ then x equals:
(A) 0 **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{9}{8}$ **(E)** $\frac{2\log_{10}3-5}{8}$

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7 The sum of the reciprocals of the roots of the equation $ax^2+bx+c=0$ is:
(A) $\frac{1}{a}+\frac{1}{b}$ **(B)** $-\frac{c}{b}$ **(C)** $\frac{b}{c}$ **(D)** $-\frac{a}{b}$ **(E)** $-\frac{b}{c}$

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8 One side of a given triangle is 18 inches. Inside the triangle a line segment is drawn parallel to this side forming a trapezoid whose area is one-third of that of the triangle. The length of this segment, in inches, is:
(A) $6\sqrt{6}$ **(B)** $9\sqrt{2}$ **(C)** 12 **(D)** $6\sqrt{3}$ **(E)** 9

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9 The vertex of the parabola $y=x^2-8x+c$ will be a point on the x-axis if the value of c is:
(A) -16 **(B)** -4 **(C)** 4 **(D)** 8 **(E)** 16

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10 The statement $x^2-x-6<0$ is equivalent to the statement:
(A) $-2<x<3$ **(B)** $x>-2$ **(C)** $x<3$ **(D)** $x>3$ and $x<-2$ **(E)** $x>3$ and $x<-2$

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11 Consider the statements: I: $(\sqrt{-4})(\sqrt{-16})=\sqrt{(-4)(-16)}$, II: $\sqrt{(-4)(-16)}=\sqrt{64}$, and $\sqrt{64}=8$.

Of these the following are incorrect.
(A) none **(B)** I only **(C)** II only **(D)** III only **(E)** I and III only

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12 A rhombus is inscribed in triangle ABC in such a way that one of its vertices is A and two of its sides lie along AB and AC. If $\overline{AC} = 6$ inches, $\overline{AB} = 12$ inches, and $\overline{BC} = 8$ inches, the side of the rhombus, in inches, is:
(A) 2 **(B)** 3 **(C)** $3\frac{1}{2}$ **(D)** 4 **(E)** 5

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13 Let n be the number of number-pairs (x, y) which satisfy $5y - 3x = 15$ and $x^2 + y^2 \leq 16$. Then n is:
(A) 0 **(B)** 1 **(C)** 2 **(D)** more than two, but finite **(E)** greater than any finite number

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14 The sum of the numerical coefficients in the complete expansion of $(x^2 - 2xy + y^2)^7$ is:
(A) 0 **(B)** 7 **(C)** 14 **(D)** 128 **(E)** 128^2

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15 The symbol 25_b represents a two-digit number in the base b. If the number 52_b is double the number 25_b, then b is:
(A) 7 **(B)** 8 **(C)** 9 **(D)** 11 **(E)** 12

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16 Let line AC be perpendicular to line CE. Connect A to D, the midpoint of CE, and connect E to B, the midpoint of AC. If AD and EB intersect in point F, and $\overline{BC} = \overline{CD} = 15$ inches, then the area of triangle DFE, in square inches, is:
(A) 50 **(B)** $50\sqrt{2}$ **(C)** 75 **(D)** $\frac{15}{2}\sqrt{105}$ **(E)** 100

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17 Given the true statement: The picnic on Sunday will not be held only if the weather is not fair. We can then conclude that:
(A) If the picnic is held, Sunday's weather is undoubtedly fair. **(B)** If the picnic is not held, Sunday's weather
(C) If it is not fair Sunday, the picnic will not be held. **(D)** If it is fair Sunday, the picnic may be held.
(E) If it is fair Sunday, the picnic must be held.

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18 If $1 - y$ is used as an approximation to the value of $\frac{1}{1+y}$, $|y| < 1$, the ratio of the error made to the correct value is:
(A) y **(B)** y^2 **(C)** $\frac{1}{1+y}$ **(D)** $\frac{y}{1+y}$ **(E)** $\frac{y^2}{1+y}$

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19 If $x^4 + 4x^3 + 6px^2 + 4qx + r$ is exactly divisible by $x^3 + 3x^2 + 9x + 3$, the value of $(p + q)r$ is:
(A) -18 **(B)** 12 **(C)** 15 **(D)** 27 **(E)** 45

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20 For every n the sum of n terms of an arithmetic progression is $2n + 3n^2$. The rth term is:
(A) $3r^2$ **(B)** $3r^2 + 2r$ **(C)** $6r - 1$ **(D)** $5r + 5$ **(E)** $6r + 2$

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21 It is possible to choose $x > \frac{2}{3}$ in such a way that the value of $\log_{10}(x^2 + 3) - 2\log_{10} x$ is
(A) negative **(B)** zero **(C)** one **(D)** smaller than any positive number that might be specified

(E) greater than any positive number that might be specified

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22 If $a_2 \neq 0$ and r and s are the roots of $a_0 + a_1x + a_2x^2 = 0$, then the equality $a_0 + a_1x + a_2x^2 = a_0\left(1 - \frac{x}{r}\right)\left(1 - \frac{x}{s}\right)$ holds:
(A) for all values of $x, a_0 \neq 0$ **(B)** for all values of x **(C)** only when $x = 0$ **(D)** only when $x = r$ or $x = s$ **(E)** only when $x = r$ or $x = s, a_0 \neq 0$

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23 If we write $|x^2 - 4| < N$ for all x such that $|x - 2| < 0.01$, the smallest value we can use for N is:
(A) $.0301$ **(B)** $.0349$ **(C)** $.0399$ **(D)** $.0401$ **(E)** $.0499$

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24 Given the sequence $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \dots, 10^{\frac{n}{11}}$, the smallest value of n such that the product of the first n members of this sequence exceeds 100000 is:
(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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25 Let $ABCD$ be a quadrilateral with AB extended to E so that $\overline{AB} = \overline{BE}$. Lines AC and CE are drawn to form angle ACE. For this angle to be a right angle it is necessary that quadrilateral $ABCD$ have:
(A) all angles equal **(B)** all sides equal **(C)** two pairs of equal sides **(D)** one pair of equal sides **(E)** one pair of equal angles

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26 For the numbers a, b, c, d, e define m to be the arithmetic mean of all five numbers; k to be the arithmetic mean of a and b; l to be the arithmetic mean of c, d, and e; and p to be the arithmetic mean of k and l. Then, no matter how a, b, c, d, and e are chosen, we shall always have:
(A) $m = p$ **(B)** $m \geq p$ **(C)** $m > p$ **(D)** $m < p$ **(E)** none of these

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27 When $y^2 + my + 2$ is divided by $y - 1$ the quotient is $f(y)$ and the remainder is R_1. When $y^2 + my + 2$ is divided by $y + 1$ the quotient is $g(y)$ and the remainder is R_2. If $R_1 = R_2$ then m is:
(A) 0 **(B)** 1 **(C)** 2 **(D)** -1 **(E)** an undetermined constant

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28 An escalator (moving staircase) of n uniform steps visible at all times descends at constant speed. Two boys, A and Z, walk down the escalator steadily as it moves, A negotiating twice as many escalator steps per minute as Z. A reaches the bottom after taking 27 steps while Z reaches the bottom after taking 18 steps. Then n is:
(A) 63 **(B)** 54 **(C)** 45 **(D)** 36 **(E)** 30

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29 Of 28 students taking at least one subject the number taking Mathematics and English only equals the number taking Mathematics only. No student takes English only or History only, and six students take Mathematics and History, but not English. The number taking English and History only is five times the number taking all three subjects. If the number taking all three subjects is even and non-zero, the number taking English and Mathematics only is:
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

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30 Let BC of right triangle ABC be the diameter of a circle intersecting hypotenuse AB in D. At D a tangent is drawn cutting leg CA in F. This information is not sufficient to prove that
(A) DF bisects CA **(B)** DF bisects $\angle CDA$ **(C)** $DF = FA$ **(D)** $\angle A = \angle BCD$ **(E)** $\angle CFD = 2\angle A$

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31 The number of real values of x satisfying the equality $(\log_2 x)(\log_b x) = \log_a b$, where $a > 0$, $b > 0$, $a \neq 1$, $b \neq 1$, is:
(A) 0 **(B)** 1 **(C)** 2 **(D)** a finite integer greater than 2 **(E)** not finite

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32 An article costing C dollars is sold for $\$100$ at a lostt of x percent of the selling price. It is then resold at a profit of x percent of the new selling price S'. If the difference between S' and C is $1\frac{1}{9}$ dollars, then x is:
(A) undetermined **(B)** $\frac{80}{9}$ **(C)** 10 **(D)** $\frac{95}{9}$ **(E)** $\frac{100}{9}$

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33 If the number $15!$, that is, $15 \cdot 14 \cdot 13 \dots 1$, ends with k zeros when given to the base 12 and ends with h zeros when given to the base 10, then $k + h$ equals:
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

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34 For $x \geq 0$ the smallest value of $\frac{4x^2+8x+13}{6(1+x)}$ is:
(A) 1 **(B)** 2 **(C)** $\frac{25}{12}$ **(D)** $\frac{13}{6}$ **(E)** $\frac{34}{5}$

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35 The length of a rectangle is 5 inches and its width is less than 4 inches. The rectangle is folded so that two diagonally opposite vertices coincide. If the length of the crease is $\sqrt{6}$, then the width is:
(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $\sqrt{5}$ **(E)** $\sqrt{\frac{11}{2}}$

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36 Given distinct straight lines OA and OB. From a point in OA a perpendicular is drawn to OB; from the foot of this perpendicular a line is drawn perpendicular to OA. From the foot of this second perpendicular a line is drawn perpendicular to OB; and so on indefinitely. The lengths of the first and second perpendiculars are a and b, respectively. Then the sum of the lengths of the perpendiculars approaches a limit as the number of perpendiculars grows beyond all bounds. This limit is:
(A) $\frac{b}{a-b}$ **(B)** $\frac{a}{a-b}$ **(C)** $\frac{ab}{a-b}$ **(D)** $\frac{b^2}{a-b}$ **(E)** $\frac{a^2}{a-b}$

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37 Point E is selected on side AB of triangle ABC in such a way that $AE : EB = 1 : 3$ and point D is selected on side BC such that $CD : DB = 1 : 2$. The point of intersection of AD and CE is F. Then $\frac{EF}{FC} + \frac{AF}{FD}$ is:
(A) $\frac{4}{5}$ **(B)** $\frac{5}{4}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** $\frac{5}{2}$

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38 A takes m times as long to do a piece of work as B and C together; B takes n times as long as C and A together; and C takes x times as long as A and B together. Then x, in terms of m and

n, is:
(A) $\frac{2mn}{m+n}$ **(B)** $\frac{1}{2(m+n)}$ **(C)** $\frac{1}{m+n-mn}$ **(D)** $\frac{1-mn}{m+n+2mn}$ **(E)** $\frac{m+n+2}{mn-1}$

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39 A foreman noticed an inspector checking a 3"-hole with a 2"-plug and a 1"-plug and suggested that two more gauges be inserted to be sure that the fit was snug. If the new gauges are alike, then the diameter, d, of each, to the nearest hundredth of an inch, is:
(A) $.87$ **(B)** $.86$ **(C)** $.83$ **(D)** $.75$ **(E)** $.71$

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40 Let n be the number of integer values of x such that $P = x^4 + 6x^3 + 11x^2 + 3x + 31$ is the square of an integer. Then n is:
(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0
