AMC 12/AHSME 1968

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1 Let P units be the increase in the circumference of a circle resulting from an increase in π units in the diameter. Then P equals:

(A) $\frac{1}{\pi}$ **(B)** π **(C)** $\frac{\pi^2}{2}$ **(D)** π^2 **(E)** 2π

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2 The real value of x such that 64^{x-1} divided by 4^{x-1} equals 256^{2x} is:

(A) $-\frac{2}{3}$ **(B)** $-\frac{1}{3}$ **(C)** 0 **(D)** $\frac{1}{4}$ **(E)** $\frac{3}{8}$

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3 A straight line passing through the point $(0, 4)$ is perpendicular to the line $x - 3y - 7 = 0$. Its equation is:

(A) $y + 3x - 4 = 0$ **(B)** $y + 3x + 4 = 0$ **(C)** $y - 3x - 4 = 0$
(D) $3y + x - 12 = 0$ **(E)** $3y - x - 12 = 0$

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4 Define an operation $*$ for positve real numbers as $a * b = \frac{ab}{a+b}$. Then $4 * (4 * 4)$ equals:

(A) $\frac{3}{4}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** 2 **(E)** $\frac{16}{3}$

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5 If $f(n) = \frac{1}{3}n(n1)(n+2)$, then $f(r) - f(r-1)$ equals:

(A) $r(r+1)$ **(B)** $(r+1)(r+2)$ **(C)** $\frac{1}{3}r(r+1)$
(D) $\frac{1}{3}(r+1)(r+2)$ **(E)** $\frac{1}{3}r(r+1)(r+2)$

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6 Let side AD of convex quadrilateral $ABCD$ be extended through D, and let side BC be extended through C, to meet in point E. Let S represent the degree-sum of angles CDE and DCE, and let S' represent the degree-sum of angles BAD and ABC. If $r = S/S'$, then:

(A) $r = 1$ sometimes, $r > 1$ sometimes
(B) $r = 1$ sometimes, $r < 1$ sometimes
(C) $0 < r < 1$ **(D)** $r > 1$ **(E)** $r = 1$

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7 Let O be the intersection point of medians AP and CQ of triangle ABC. If OQ is 3 inches, then OP, in inches, is:

(A) 3 **(B)** $\frac{9}{2}$ **(C)** 6 **(D)** 9 **(E)** undetermined

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8 A positive number is mistakenly divided by 6 instead of being multiplied by 6. Based on the correct answer, the error thus comitted, to the nearest percent, is:

(A) 100 **(B)** 97 **(C)** 83 **(D)** 17 **(E)** 3

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9 The sum of the real values of x satisfying the equality $|x+2| = 2|x-2|$ is:

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** 6 **(D)** $6\frac{1}{3}$ **(E)** $6\frac{2}{3}$

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10 Assume that, for a certain school, it is true that
I: Some students are not honest
II: All fraternity members are honest
A necessary conclusion is:

(A) Some students are fraternity members
(B) Some fraternity members are not students
(C) Some students are not fraternity members
(D) No fraternity member is a student
(E) No student is a fraternity member

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11 If an arc of 60° on circle I has the same length as an arc of 45° on circle II, the ratio of the area of circle I to that of circle II is:

(A) $16:9$ **(B)** $9:16$ **(C)** $4:3$ **(D)** $3:4$ **(E)** None of these

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12 A circle passes through the vertices of a triangle with side-lengths of $7\tfrac{1}{2}, 10, 12\tfrac{1}{2}$. The radius of the circle is:

(A) $\frac{15}{4}$ **(B)** 5 **(C)** $\frac{25}{4}$ **(D)** $\frac{35}{4}$ **(E)** $\frac{15\sqrt{2}}{2}$

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13 If m and n are the roots of $x^2+mx+n=0$, $m \neq 0$, $n \neq 0$, then the sum of the roots is:

(A) $-\frac{1}{2}$ **(B)** -1 **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** Undetermined

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14 If x and y are non-zero numbers such that $x = 1 + \frac{1}{y}$ and $y = 1 + \frac{1}{x}$, then y equals:

(A) $x-1$ **(B)** $1-x$ **(C)** $1+x$ **(D)** $-x$ **(E)** x

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15 Let P be the product of any three consecutive positive odd integers. The largest integer dividing all such P is:

(A) 15 **(B)** 6 **(C)** 5 **(D)** 3 **(E)** 1

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16 If x is such that $\frac{1}{x} < 2$ and $\frac{1}{x} > -3$, then:

(A) $-\frac{1}{3} < x < \frac{1}{2}$ **(B)** $-\frac{1}{2} < x < 3$ **(C)** $x > \frac{1}{2}$
(D) $x > \frac{1}{2}$ or $-\frac{1}{3} < x < 0$ **(E)** $x > \frac{1}{2}$ or $x < -\frac{1}{3}$

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17 Let $f(n) = \frac{x_1 + x_2 + ... + x_n}{n}$, where n is a positive integer. If $x_k = (-1)^k, k = 1, 2, ..., n$, the set of possible values of $f(n)$ is:

(A) $\{0\}$ **(B)** $\{\frac{1}{n}\}$ **(C)** $\{0, -\frac{1}{n}\}$ **(D)** $\{0, \frac{1}{n}\}$ **(E)** $\{1, \frac{1}{n}\}$

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18 Side AB of triangle ABC has length 8 inches. Line DEF is drawn parallel to AB so that D is on segment AC, and E is on segment BC. Line AE extended bisects angle FEC. If DE has length 5 inches, then the length of CE, in inches, is:

(A) $\frac{51}{4}$ **(B)** 13 **(C)** $\frac{53}{4}$ **(D)** $\frac{40}{3}$ **(E)** $\frac{27}{2}$

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19 Let n be the number of ways that 10 dollars can be changed into dimes and quarters, with at least one of each coin being used. Then n equals:

(A) 40 **(B)** 38 **(C)** 21 **(D)** 20 **(E)** 19

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20 The measures of the interior angles of a convex polygon of n sides are in arithmetic progression. If the common difference is $5°$ and the largest angle is $160°$, then n equals:

(A) 9 **(B)** 10 **(C)** 12 **(D)** 16 **(E)** 32

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21 If $S = 1! + 2! + 3! + \cdots + 99!$, then the units' digit in the value of S is:

(A) 9 **(B)** 8 **(C)** 5 **(D)** 3 **(E)** 0

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22 A segment of length 1 is divided into four segments. Then there exists a quadrilateral with the four segments as sides if and only if each segment is:

(A) equal to $\frac{1}{4}$
(B) equal to or greater than $\frac{1}{8}$ and less than $\frac{1}{2}$
(C) greater than $\frac{1}{8}$ and less than $\frac{1}{2}$
(D) greater than $\frac{1}{8}$ and less than $\frac{1}{4}$
(E) less than $\frac{1}{2}$

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23 If all the logarithms are real numbers, the equality

$$\log(x + 3) + \log(x - 1) = \log(x^2 - 2x - 3)$$

is satisfied for:

(A) all real values of x
(B) no real values of x
(C) all real values of x except $x = 0$
(D) no real values of x except $x = 0$
(E) all real values of x except $x = 1$

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24 A painting $18''$ X $24''$ is to be placed into a wooden frame with the longer dimension vertical. The wood at the top and bottom is twice as wide as the wood on the sides. If the frame area equals that of the painting itself, the ratio of the smaller to the larger dimension of the framed painting is:

(A) $1:3$ **(B)** $1:2$ **(C)** $2:3$ **(D)** $3:4$ **(E)** $1:1$

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25 Ace runs with constant speed and Flash runs x times as fast, $x > 1$. Flash gives Ace a head start of y yards, and, at a given signal, they start off in the same direction. Then the number of yards Flash must run to catch Ace is:

(A) xy **(B)** $\frac{y}{x+y}$ **(C)** $\frac{xy}{x-1}$ **(D)** $\frac{x+y}{x+1}$ **(E)** $\frac{x+y}{x-1}$

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26 Let $S = 2 + 4 + 6 + \cdots + 2N$, where N is the smallest positive integer such that $S > 1,000,000$. Then the sum of the digits of N is:

(A) 27 **(B)** 12 **(C)** 6 **(D)** 2 **(E)** 1

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27 Let $S_n = 1 - 2 + 3 - 4 + \cdots + (-1)^{n-1}n,\ n = 1, 2, \cdots$. Then $S_{17} + S_{33} + S_{50}$ equals:

(A) 0 **(B)** 1 **(C)** 2 **(D)** -1 **(E)** -2

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28 If the arithmetic mean of a and b is double their geometric mean, with $a > b > 0$, then a possible value for the ratio $\frac{a}{b}$, to the nearest integer, is

(A) 5 **(B)** 8 **(C)** 11 **(D)** 14 **(E)** none of these

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29 Given the three numbers $x, y = x^x, z = x^{(x^x)}$ with $.9 < x < 1.0$. Arranged in order of increasing magnitude, they are:

(A) x, z, y **(B)** x, y, z **(C)** y, x, z **(D)** y, z, x **(E)** z, x, y

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30 Convex polygons P_1 and P_2 are drawn in the same plane with n_1 and n_2 sides, respectively, $n_1 \le n_2$. If P_1 and P_2 do not have any line segment in common, then the maximum number of intersections of P_1 and P_2 is:

(A) $2n_1$ **(B)** $2n_2$ **(C)** $n_1 n_2$ **(D)** $n_1 + n_2$ **(E)** none of these

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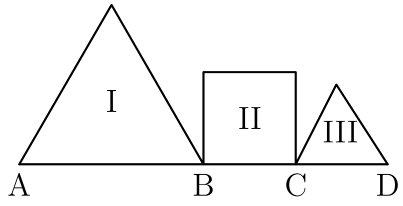
31 In this diagram, not drawn to scale, figures I and III are equilateral triangular regions with respective areas of $32\sqrt{3}$ and $8\sqrt{3}$ square inches. Figure II is a square region with area 32 sq. in. Let the length of segment AD be decreased by $12\frac{1}{2}\%$ of itself, while the lengths of AB and CD remain unchanged. The percent decrease in the area of the square is:



(A) $12\frac{1}{2}$ **(B)** 25 **(C)** 50 **(D)** 75 **(E)** $87\frac{1}{2}$

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32 A and B move uniformly along two straight paths intersecting at right angles in point O. When A is at O, B is 500 yards short of O. In 2 minutes, they are equidistant from O, and in 8 minutes more they are again equidistant from O. Then the ratio of A's speed to B's speed is:

(A) $4:5$ **(B)** $5:6$ **(C)** $2:3$ **(D)** $5:8$ **(E)** $1:2$

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33 A number N has three digits when expressed in base 7. When N is expressed in base 9 the digits are reversed. Then the middle digit is:

(A) 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** 5

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34 With 400 members voting the House of Representatives defeated a bill. A re-vote, with the same members voting, resulted in passage of the bill by twice the margin† by which it was originally defeated. The number voting for the bill on the re-vote was $\frac{12}{11}$ of the number voting against it originally. How many more members voted for the bill the second time than voted for it the first time?

(A) 75 **(B)** 60 **(C)** 50 **(D)** 45 **(E)** 20

† In this context, margin of defeat (passage) is defined as the number of nays minus the number of ayes (nays-ayes).

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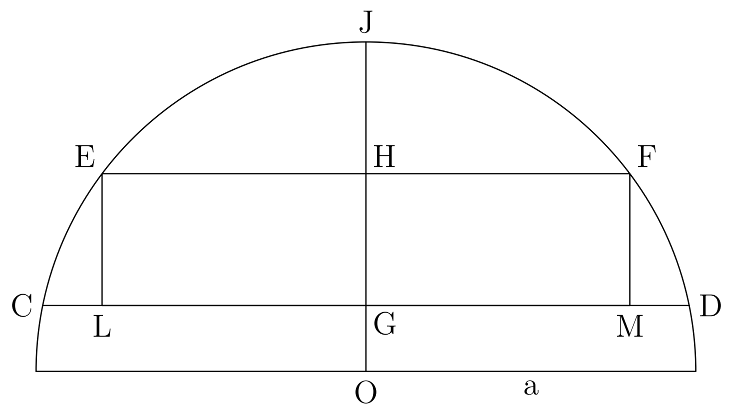
35 In this diagram the center of the circle is O, the radius is a inches, chord EF is parallel to chord CD, O, G, H, J are collinear, and G is the midpoint of CD. Let K (sq. in.) represent the area of trapezoid $CDFE$ and let R (sq. in.) represent the area of rectangle $ELMF$. Then, as CD and EF are translated upward so that OG increases toward the value a, while JH always equals HG, the ratio $K:R$ become arbitrarily close to:



(A) 0 **(B)** 1 **(C)** $\sqrt{2}$ **(D)** $\frac{1}{\sqrt{2}}+\frac{1}{2}$ **(E)** $\frac{1}{\sqrt{2}}+1$