AMC 12/AHSME 2008

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by worthawholebean, mathgeniuse^ln(x), Altheman, infinity4ever, E^(pi*i)=-1, Xantos C. Guin, azjps, MellowMelon, archimedes1, TZF, andersonw, nickster08, 123456789, randomdragoon, n0vad3m0n, krsattack, CatalystOfNostalgia, rrusczyk

– A

– February 10th

1 A bakery owner turns on his doughnut machine at 8:30 AM. At 11:10 AM the machine has completed one third of the day's job. At what time will the doughnut machine complete the job?
(A) 1:50 PM **(B)** 3:00 PM **(C)** 3:30 PM **(D)** 4:30 PM **(E)** 5:50 PM

2 What is the reciprocal of $\frac{1}{2}+\frac{2}{3}$?
(A) $\frac{6}{7}$ **(B)** $\frac{7}{6}$ **(C)** $\frac{5}{3}$ **(D)** 3 **(E)** $\frac{7}{2}$

3 Suppose that $\frac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much is $\frac{1}{2}$ of 5 bananas?
(A) 2 **(B)** $\frac{5}{2}$ **(C)** 3 **(D)** $\frac{7}{2}$ **(E)** 4

4 Which of the following is equal to the product

$$\frac{8}{4}\cdot\frac{12}{8}\cdot\frac{16}{12}\cdots\frac{4n+4}{4n}\cdots\frac{2008}{2004}?$$

(A) 251 **(B)** 502 **(C)** 1004 **(D)** 2008 **(E)** 4016

5 Suppose that

$$\frac{2x}{3}-\frac{x}{6}$$

is an integer. Which of the following statements must be true about x?
(A) It is negative. **(B)** It is even, but not necessarily a multiple of 3. **(C)** It is a multiple of 3, but not neces
(D) It is a multiple of 6, but not necessarily a multiple of 12. **(E)** It is a multiple of 12.

6 Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a $\$90$ rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves $\$15$ by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?
(A) 750 **(B)** 900 **(C)** 1000 **(D)** 1050 **(E)** 1500

7 While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?
(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

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8 What is the volume of a cube whose surface area is twice that of a cube with volume 1?
(A) $\sqrt{2}$ **(B)** 2 **(C)** $2\sqrt{2}$ **(D)** 4 **(E)** 8

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9 Older television screens have an aspect ratio of $4:3$. That is, the ratio of the width to the height is $4:3$. The aspect ratio of many movies is not $4:3$, so they are sometimes shown on a television screen by 'letterboxing' - darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of $2:1$ and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?

(A) 2 **(B)** 2.25 **(C)** 2.5 **(D)** 2.7 **(E)** 3

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10 Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t?
(A) $\left(\frac{1}{5}+\frac{1}{7}\right)(t+1)=1$ **(B)** $\left(\frac{1}{5}+\frac{1}{7}\right)t+1=1$ **(C)** $\left(\frac{1}{5}+\frac{1}{7}\right)t=1$
(D) $\left(\frac{1}{5}+\frac{1}{7}\right)(t-1)=1$ **(E)** $(5+7)t=1$

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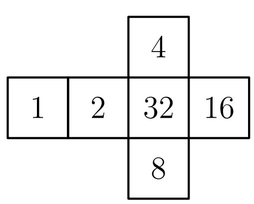
11 Three cubes are each formed from the pattern shown. They are then stacked on a table one on top of another so that the 13 visible numbers have the greatest possible sum. What is that sum?



(A) 154 **(B)** 159 **(C)** 164 **(D)** 167 **(E)** 189

12 A function f has domain $[0,2]$ and range $[0,1]$. (The notation $[a,b]$ denotes $\{x : a \le x \le b\}$.) What are the domain and range, respectively, of the function g defined by $g(x) = 1 - f(x+1)$?
(A) $[-1,1],[-1,0]$ **(B)** $[-1,1],[0,1]$ **(C)** $[0,2],[-1,0]$ **(D)** $[1,3],[-1,0]$ **(E)** $[1,3],[0,1]$

13 Points A and B lie on a circle centered at O, and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

14 What is the area of the region defined by the inequality $|3x-18|+|2y+7| \le 3$?
(A) 3 **(B)** $\frac{7}{2}$ **(C)** 4 **(D)** $\frac{9}{2}$ **(E)** 5

15 Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?
(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

16 The numbers $\log(a^3b^7)$, $\log(a^5b^{12})$, and $\log(a^8b^{15})$ are the first three terms of an arithmetic sequence, and the 12^{th} term of the sequence is $\log b^n$. What is n?
(A) 40 **(B)** 56 **(C)** 76 **(D)** 112 **(E)** 143

17 Let $a_1, a_2, \ldots$ be a sequence of integers determined by the rule $a_n = a_{n-1}/2$ if a_{n-1} is even and $a_n = 3a_{n-1}+1$ if a_{n-1} is odd. For how many positive integers $a_1 \le 2008$ is it true that a_1 is less than each of a_2, a_3, and a_4?
(A) 250 **(B)** 251 **(C)** 501 **(D)** 502 **(E)** 1004

18 Triangle ABC, with sides of length 5, 6, and 7, has one vertex on the positive x-axis, one on the positive y-axis, and one on the positive z-axis. Let O be the origin. What is the volume of tetrahedron $OABC$?
(A) $\sqrt{85}$ **(B)** $\sqrt{90}$ **(C)** $\sqrt{95}$ **(D)** 10 **(E)** $\sqrt{105}$

19 In the expansion of

$$\left(1+x+x^2+\cdots+x^{27}\right)\left(1+x+x^2+\cdots+x^{14}\right)^2,$$

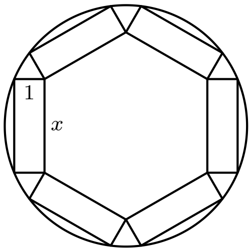
what is the coefficient of x^{28}?
(A) 195 **(B)** 196 **(C)** 224 **(D)** 378 **(E)** 405

20 Triangle ABC has $AC=3$, $BC=4$, and $AB=5$. Point D is on $\overline{AB}$, and $\overline{CD}$ bisects the right angle. The inscribed circles of $\triangle ADC$ and $\triangle BCD$ have radii r_a and r_b, respectively. What is r_a/r_b?
(A) $\frac{1}{28}\left(10-\sqrt{2}\right)$ **(B)** $\frac{3}{56}\left(10-\sqrt{2}\right)$ **(C)** $\frac{1}{14}\left(10-\sqrt{2}\right)$ **(D)** $\frac{5}{56}\left(10-\sqrt{2}\right)$
(E) $\frac{3}{28}\left(10-\sqrt{2}\right)$

21 A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?

(A) 36 **(B)** 40 **(C)** 44 **(D)** 48 **(E)** 52

22 A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x. Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?



(A) $2\sqrt{5}-\sqrt{3}$ **(B)** 3 **(C)** $\frac{3\sqrt{7}-\sqrt{3}}{2}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{5+2\sqrt{3}}{2}$

23 The solutions of the equation $z^4 + 4z^3i - 6z^2 - 4zi - i = 0$ are the vertices of a convex polygon in the complex plane. What is the area of the polygon?

(A) $2^{5/8}$ **(B)** $2^{3/4}$ **(C)** 2 **(D)** $2^{5/4}$ **(E)** $2^{3/2}$

24 Triangle ABC has $\angle C = 60^\circ$ and $BC = 4$. Point D is the midpoint of BC. What is the largest possible value of $\tan \angle BAD$?

(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{\sqrt{3}}{2\sqrt{2}}$ **(D)** $\frac{\sqrt{3}}{4\sqrt{2}-3}$ **(E)** 1

25 A sequence (a_1, b_1), (a_2, b_2), (a_3, b_3), $\ldots$ of points in the coordinate plane satisfies

$$(a_{n+1}, b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n) \quad \text{for} \quad n = 1, 2, 3, \ldots.$$

Suppose that $(a_{100}, b_{100}) = (2, 4)$. What is $a_1 + b_1$?

(A) $\frac{1}{2^{97}}$ **(B)** $\frac{1}{2^{99}}$ **(C)** 0 **(D)** $\frac{1}{2^{98}}$ **(E)** $\frac{1}{2^{96}}$

– B

– February 27th

1 A basketball player made 5 baskets during a game. Each basket was worth either 2 or 3 points. How many different numbers could represent the total points scored by the player?
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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2 A 4×4 block of calendar dates is shown. The order of the numbers in the second row is to be reversed. Then the order of the numbers in the fourth row is to be reversed. Finally, the numbers on each diagonal are to be added. What will be the positive difference between the two diagonal sums?

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

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3 A semipro baseball league has teams with 21 players each. League rules state that a player must be paid at least $\$15,000$, and that the total of all players' salaries for each team cannot exceed $\$700,000$. What is the maximum possiblle salary, in dollars, for a single player?
(A) $270,000$ **(B)** $385,000$ **(C)** $400,000$ **(D)** $430,000$ **(E)** $700,000$

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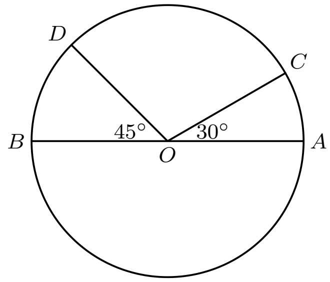
4 On circle O, points C and D are on the same side of diameter $\overline{AB}$, $\angle AOC = 30^\circ$, and $\angle DOB = 45^\circ$. What is the ratio of the area of the smaller sector COD to the area of the circle?



(A) $\frac{2}{9}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{7}{24}$ **(E)** $\frac{3}{10}$

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5 A class collects $\$50$ to buy flowers for a classmate who is in the hospital. Roses cost $\$3$ each, and carnations cost $\$2$ each. No other flowers are to be used. How many different bouquets could be purchased for exactly $\$50$?
(A) 1 **(B)** 7 **(C)** 9 **(D)** 16 **(E)** 17

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6 Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to

00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year?
(A) 2500 **(B)** 3000 **(C)** 3500 **(D)** 4000 **(E)** 4500

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7 For real numbers a and b, define $a\$b = (a-b)^2$. What is $(x-y)^2\$(y-x)^2$?
(A) 0 **(B)** x^2+y^2 **(C)** $2x^2$ **(D)** $2y^2$ **(E)** $4xy$

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8 Points B and C lie on $\overline{AD}$. The length of $\overline{AB}$ is 4 times the length of $\overline{BD}$, and the length of $\overline{AC}$ is 9 times the length of $\overline{CD}$. The length of $\overline{BC}$ is what fraction of the length of $\overline{AD}$?
(A) $\frac{1}{36}$ **(B)** $\frac{1}{13}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{5}{36}$ **(E)** $\frac{1}{5}$

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9 Points A and B are on a circle of radius 5 and $AB = 6$. Point C is the midpoint of the minor arc AB. What is the length of the line segment AC?
(A) $\sqrt{10}$ **(B)** $\frac{7}{2}$ **(C)** $\sqrt{14}$ **(D)** $\sqrt{15}$ **(E)** 4

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10 Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?
(A) 500 **(B)** 900 **(C)** 950 **(D)** 1000 **(E)** 1900

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11 A cone-shaped mountain has its base on the ocean floor and has a height of 8000 feet. The top $\frac{1}{8}$ of the volume of the mountain is above water. What is the depth of the ocean at the base of the mountain, in feet?
(A) 4000 **(B)** $2000(4-\sqrt{2})$ **(C)** 6000 **(D)** 6400 **(E)** 7000

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12 For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?
(A) 2008 **(B)** 4015 **(C)** 4016 **(D)** $4,030,056$ **(E)** $4,032,064$

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13 Vertex E of equilateral $\triangle ABE$ is in the interior of unit square $ABCD$. Let R be the region consisting of all points inside $ABCD$ and outside $\triangle ABE$ whose distance from $\overline{AD}$ is between $\frac{1}{3}$ and $\frac{2}{3}$. What is the area of R?
(A) $\frac{12-5\sqrt{3}}{72}$ **(B)** $\frac{12-5\sqrt{3}}{36}$ **(C)** $\frac{\sqrt{3}}{18}$ **(D)** $\frac{3-\sqrt{3}}{9}$ **(E)** $\frac{\sqrt{3}}{12}$

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14 A circle has a radius of $\log_{10}(a^2)$ and a circumference of $\log_{10}(b^4)$. What is $\log_a b$?
(A) $\frac{1}{4\pi}$ **(B)** $\frac{1}{\pi}$ **(C)** π **(D)** 2π **(E)** $10^{2\pi}$

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15 On each side of a unit square, an equilateral triangle of side length 1 is constructed. On each new side of each equilateral triangle, another equilateral triangle of side length 1 is constructed. The interiors of the square and the 12 triangles have no points in common. Let R be the region formed by the union of the square and all the triangles, and S be the smallest convex polygon

that contains R. What is the area of the region that is inside S but outside R?
(A) $\frac{1}{4}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** 1 **(D)** $\sqrt{3}$ **(E)** $2\sqrt{3}$

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16 A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

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17 Let A, B, and C be three distinct points on the graph of $y = x^2$ such that line AB is parallel to the x-axis and $\triangle ABC$ is a right triangle with area 2008. What is the sum of the digits of the y-coordinate of C?
(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

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18 A pyramid has a square base $ABCD$ and vertex E. The area of square $ABCD$ is 196, and the areas of $\triangle ABE$ and $\triangle CDE$ are 105 and 91, respectively. What is the volume of the pyramid?
(A) 392 **(B)** $196\sqrt{6}$ **(C)** $392\sqrt{2}$ **(D)** $392\sqrt{3}$ **(E)** 784

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19 A function f is defined by $f(z) = (4+i)z^2 + \alpha z + \gamma$ for all complex numbers z, where α and γ are complex numbers and $i^2 = -1$. Suppose that $f(1)$ and $f(i)$ are both real. What is the smallest possible value of $|\alpha| + |\gamma|$?
(A) 1 **(B)** $\sqrt{2}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** 4

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20 Michael walks at the rate of 5 feet per second on a long straight path. Trash pails are located every 200 feet along the path. A garbage truck travels at 10 feet per second in the same direction as Michael and stops for 30 seconds at each pail. As Michael passes a pail, he notices the truck ahead of him just leaving the next pail. How many times will Michael and the truck meet?
(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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21 Two circles of radius 1 are to be constructed as follows. The center of circle A is chosen uniformly and at random from the line segment joining $(0, 0)$ and $(2, 0)$. The center of circle B is chosen uniformly and at random, and independently of the first choice, from the line segment joining $(0, 1)$ to $(2, 1)$. What is the probability that circles A and B intersect?
(A) $\frac{2+\sqrt{2}}{4}$ **(B)** $\frac{3\sqrt{3}+2}{8}$ **(C)** $\frac{2\sqrt{2}-1}{2}$ **(D)** $\frac{2+\sqrt{3}}{4}$ **(E)** $\frac{4\sqrt{3}-3}{4}$

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22 A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?
(A) $\frac{11}{20}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{81}{140}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{17}{28}$

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23 The sum of the base-10 logarithms of the divisors of 10^n is 792. What is n?
(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

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24 Let $A_0 = (0, 0)$. Distinct points $A_1, A_2, \ldots$ lie on the x-axis, and distinct points $B_1, B_2, \ldots$ lie on the graph of $y = \sqrt{x}$. For every positive integer n, $A_{n-1}B_nA_n$ is an equilateral triangle. What is the least n for which the length $A_0A_n \geq 100$?
(A) 13 **(B)** 15 **(C)** 17 **(D)** 19 **(E)** 21

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25 Let $ABCD$ be a trapezoid with $AB||CD$, $AB = 11$, $BC = 5$, $CD = 19$, and $DA = 7$. Bisectors of $\angle A$ and $\angle D$ meet at P, and bisectors of $\angle B$ and $\angle C$ meet at Q. What is the area of hexagon $ABQCDP$?
(A) $28\sqrt{3}$ **(B)** $30\sqrt{3}$ **(C)** $32\sqrt{3}$ **(D)** $35\sqrt{3}$ **(E)** $36\sqrt{3}$
