AMC 12/AHSME 2023

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–	A
–	November 8, 2023
1	Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet? **(A)** 20 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27
2	The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza? **(A)** $1\frac{4}{5}$ **(B)** 2 **(C)** $2\frac{2}{5}$ **(D)** 3 **(E)** $3\frac{3}{5}$
3	How many positive perfect squares less than 2023 are divisible by 5? **(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12
4	How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$? **(A)** 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18
5	Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3? **(A)** $\frac{2}{9}$ **(B)** $\frac{49}{216}$ **(C)** $\frac{25}{108}$ **(D)** $\frac{17}{72}$ **(E)** $\frac{13}{54}$
6	Points A and B lie on the graph of $y=\log_2 x$. The midpoint of $\overline{AB}$ is $(6,2)$. What is the positive difference between the x-coordinates of A and B? **(A)** $2\sqrt{11}$ **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** $4\sqrt{5}$ **(E)** 9
7	A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 will each digit appear an even number of times in the 8-digital display for that date?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

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8 Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?
(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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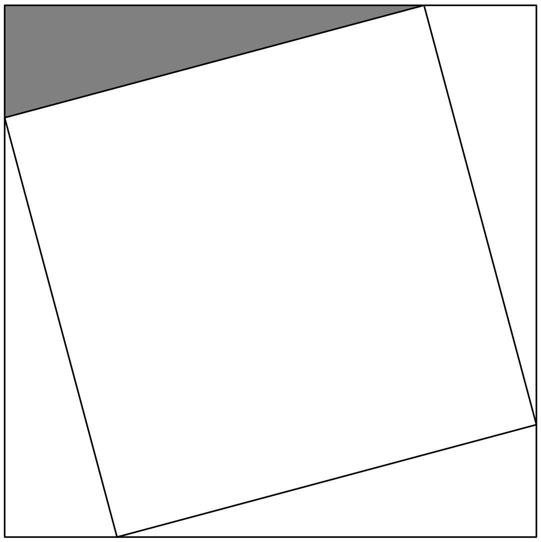
9 A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?



(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\sqrt{3}-\sqrt{2}$ **(E)** $\sqrt{2}-1$

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10 Positive real numbers x and y satisfy $y^3=x^2$ and $(y-x)^2=4y^2$. What is $x+y$?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 42

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11 What is the degree measure of the acute angle formed by lines with slopes 2 and $\frac{1}{3}$?

(A) 30 **(B)** 37.5 **(C)** 45 **(D)** 52.5 **(E)** 60

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12 What is the value of

$$2^3-1^2+4^3-3^3+6^3-5^3+\cdots+18^3-17^3?$$

(A) 2023 **(B)** 2679 **(C)** 2941 **(D)** 3159 **(E)** 3235

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13 In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

(A) 15 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 66

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14 How many complex numbers satisfy the equation $z^5 = \overline{z}$, where $\overline{z}$ is the conjugate of the complex number z?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

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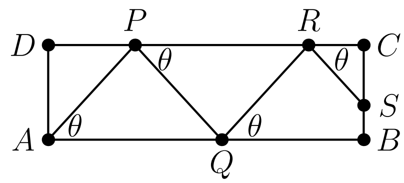
15 Usain is walking for exercise by zigzagging across a 100-meter by 30-meter rectangular field, beginning at point A and ending on the segment $\overline{BC}$. He wants to increase the distance walked by zigzagging as shown in the figure below $(APQRS)$. What angle $\theta = \angle PAB = \angle QPC = \angle RQB = \cdots$ will produce in a length that is 120 meters? (Do not assume the zigzag path has exactly four segments as shown; there could be more or fewer.)



(A) $\arccos\frac{5}{6}$ **(B)** $\arccos\frac{4}{5}$ **(C)** $\arccos\frac{3}{10}$ **(D)** $\arcsin\frac{4}{5}$ **(E)** $\arcsin\frac{5}{6}$

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16 Consider the set of complex numbers z satisfying $|1+z+z^2| = 4$. The maximum value of the imaginary part of z can be written in the form $\frac{\sqrt{m}}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

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17 Flora the frog starts at 0 on the number line and makes a sequence of jumps to the right. In any one jump, independent of previous jumps, Flora leaps a positive integer distance m with probability $\frac{1}{2^m}$. What is the probability that Flora will eventually land at 10?

(A) $\frac{5}{512}$ **(B)** $\frac{45}{1024}$ **(C)** $\frac{127}{1024}$ **(D)** $\frac{511}{1024}$ **(E)** $\frac{1}{2}$

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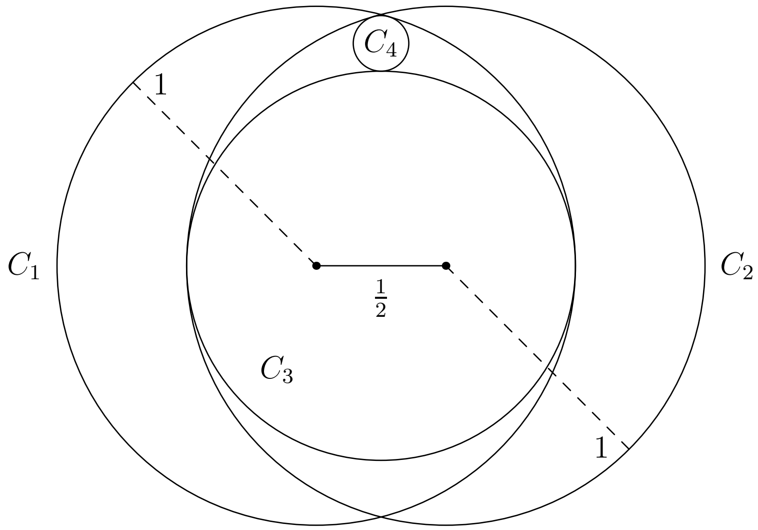
18 Circle C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4?



(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

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19 What is the product of all the solutions to the equation

$$\log_{7x} 2023 \cdot \log_{289x} 2023 = \log_{2023x} 2023?$$

(A) $(\log_{2023} 7 \cdot \log_{2023} 289)^2$ **(B)** $\log_{2023} 7 \cdot \log_{2023} 289$ **(C)** 1 **(D)** $\log_7 2023 \cdot \log_{289} 2023$ **(E)** $(\log_7 20$
$\log_{289} 2023)^2$

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20 Rows 1, 2, 3, 4, and 5 of a triangular array of integers are shown below:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 3 & & 1 & & \\ & 1 & & 5 & & 5 & & 1 & \\ 1 & & 7 & & 11 & & 7 & & 1 \end{array}$$

Each row after the first row is formed by placing a 1 at each end of the row, and each interior entry is 1 greater than the sum of the two numbers diagonally above it in the previous row. What is the units digit of the sum of the 2023 numbers in the 2023rd row?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

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21 If A and B are vertices of a polyhedron, define the *distance* $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$

is an edge of the polyhedron, then $d(A, B) = 1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

(A) $\frac{7}{22}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{1}{2}$

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22 Let f be the unique function defined on the positive integers such that

$$\sum_{d|n} d \cdot f\left(\frac{n}{d}\right) = 1$$

for all positive integers n, where the sum is taken over all positive divisors of n. What is $f(2023)$?

(A) -1536 **(B)** 96 **(C)** 108 **(D)** 116 **(E)** 144

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23 How many ordered pairs of positive real numbers (a, b) satisfy the equation

$$(1 + 2a)(2 + 2b)(2a + b) = 32ab?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** an infinite number

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24 Let K be the number of sequences A_1, A_2, $\dots$, A_n such that n is a positive integer less than or equal to 10, each A_i is a subset of $\{1, 2, 3, \dots, 10\}$, and A_{i-1} is a subset of A_i for each i between 2 and n, inclusive. For example, $\{\}$, $\{5, 7\}$, $\{2, 5, 7\}$, $\{2, 5, 7\}$, $\{2, 5, 6, 7, 9\}$ is one such sequence, with $n = 5$. What is the remainder when K is divided by 10?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

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25 There is a unique sequence of integers $a_1, a_2, \cdots a_{2023}$ such that

$$\tan 2023x = \frac{a_1 \tan x + a_3 \tan^3 x + a_5 \tan^5 x + \cdots + a_{2023} \tan^{2023} x}{1 + a_2 \tan^2 x + a_4 \tan^4 x \cdots + a_{2022} \tan^{2022} x}$$

whenever $\tan 2023x$ is defined. What is a_{2023}?

(A) -2023 **(B)** -2022 **(C)** -1 **(D)** 1 **(E)** 2023

– B

– November 14, 2023

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1 Mrs. Jones is pouring orange juice for her 4 kids into 4 identical glasses. She fills the first 3 full, but only has enough orange juice to fill one third of the last glass. What fraction of a glass of orange juice does she need to pour from the 3 full glasses into the last glass so that all glasses have an equal amount of orange juice? **(A)** $\frac{1}{12}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{2}{9}$

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2 Carlos went to a sports store to buy running shoes. Running shoes were on sale, with prices reduced by 20

A)46 B)50 C)48 D)47 E)49

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3 A $3-4-5$ right triangle is inscribed in circle A, and a $5-12-13$ right triangle is inscribed in circle B. What is the ratio of the area of circle A to the area of circle B?

(A) $\frac{9}{25}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{25}{169}$ **(E)** $\frac{4}{25}$

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4 Jackson's paintbrush makes a narrow strip that is 6.5 mm wide. Jackson has enough paint to make a strip of 25 meters. How much can he paint, in cm^2? **(A)** 16.25 **(B)** 162.5 **(C)** 1625 **(D)** 1625
(These aren't in the right order)

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5 You are playing a game. A 2×1 rectangle covers two adjacent squares (oriented either horizontally or vertically) of a 3×3 grid of squares, but you are not told which two squares are covered. Your goal is to find at least one square that is covered by the rectangle. A "turn" consists of you guessing a square, after which you are told whether that square is covered by the hidden rectangle. What is the minimum number of turns you need to ensue that at least one of your guessed squares is covered by the rectangle?

(A) 3 **(B)** 5 **(C)** 4 **(D)** 8 **(E)** 6

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6 When the roots of the polynomial

$$P(x) = \prod_{i=1}^{10} (x-i)^i$$

are removed from the real number line, what remains is the union of 11 disjoint open intervals. On how many of those intervals is $P(x)$ positive?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

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7 For how many integers n does the expression

$$\sqrt{\frac{\log(n^2) - (\log n)^2}{\log n - 3}}$$

represent a real number, where log denotes the base 10 logarithm? **(A)** 900 **(B)** 2 **(C)** 902 **(D)** 2

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8 How many nonempty subsets B of $\{0, 1, 2, 3, \ldots, 12\}$ have the property that the number of elements in B is equal to the least element of B? For example, $B = \{4, 6, 8, 11\}$ satisfies the condition.

(A) 256 **(B)** 136 **(C)** 108 **(D)** 144 **(E)** 156

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9 What is the area of the region in the coordinate plane defined by the inequality

$$||x|-1|+||y|-1| \leq 1?$$

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

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10 In the xy-plane, a circle of radius 4 with center on the positive x-axis is tangent to the y-axis at the origin, and a circle with radius 10 with center on the positive y-axis is tangent to the x-axis at the origin. What is the slope of the line passing through the two points at which these circles intersect?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{2}{\sqrt{29}}$ **(D)** $\frac{1}{\sqrt{29}}$ **(E)** $\frac{2}{5}$

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11 What is the maximum area of an isosceles trapezoid that has legs of length 1 and one base twice as long as the other? **(A)** $\frac{5}{4}$ **(B)** $\frac{8}{7}$ **(C)** $\frac{5\sqrt{2}}{4}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{3\sqrt{3}}{4}$

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12 For complex numbers $u=a+bi$ and $v=c+di$, define the binary operation $\otimes$ by

$$u \otimes v = ac + bdi.$$

Suppose z is a complex number such that $z \otimes z = z^2 + 40$. What is $|z|$?

(A) $\sqrt{10}$ **(B)** $3\sqrt{2}$ **(C)** $2\sqrt{6}$ **(D)** 6 **(E)** $5\sqrt{2}$

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13 A rectangular box $\mathcal{P}$ has distinct edge lengths a, b, and c. The sum of the lengths of all 12 edges of $\mathcal{P}$ is 13, the sum of the areas of all 6 faces of $\mathcal{P}$ is $\frac{11}{2}$, and the volume of $\mathcal{P}$ is $\frac{1}{2}$. What is the length of the longest interior diagonal connecting two vertices of $\mathcal{P}$?

(A) 2 **(B)** $\frac{3}{8}$ **(C)** $\frac{9}{8}$ **(D)** $\frac{9}{4}$ **(E)** $\frac{3}{2}$

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14 For how many ordered pairs (a,b) of integers does the polynomial x^3+ax^2+bx+6 have 3 distinct integer roots?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 7 **(E)** 4

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15 Suppose a, b, and c are positive integers such that

$$\frac{a}{14}+\frac{b}{15}=\frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a,14)=1$ or $\gcd(b,15)=1$ or both, then $\gcd(c,210)=1$.
II. If $\gcd(c,210)=1$, then $\gcd(a,14)=1$ or $\gcd(b,15)=1$ or both.
III. $\gcd(c,210)=1$ if and only if $\gcd(a,14)=\gcd(b,15)=1$.

(A) I, II, and III **(B)** I only **(C)** I and II only **(D)** III only **(E)** II and III only

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16 In Coinland, there are three types of coins, each worth $6, 10,$ and 15. What is the sum of the digits of the maximum amount of money that is impossible to have?

(A) 11 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10
(I forgot the order)

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17 Triangle ABC has side lengths in arithmetic progression, and the smallest side has length 6. If the triangle has an angle of $120°$, what is the area of ABC?

(A) $12\sqrt{3}$ **(B)** $8\sqrt{6}$ **(C)** $14\sqrt{2}$ **(D)** $20\sqrt{2}$ **(E)** $15\sqrt{3}$

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18 Last academic year Yolanda and Zelda took different courses that did not necessarily administer the same number of quizzes during each of the two semesters. Yolanda's average on all the quizzes she took during the first semester was 3 points higher than Zelda's average on all the quizzes she took during the first semester. Yolanda's average on all the quizzes she took during the second semester was 18 points higher than her average for the first semester and was again 3 points higher than Zelda's average on all the quizzes Zelda took during her second semester. Which one of the following statements cannot possibly be true?

(A) Yolanda's quiz average for the academic year was 22 points higher than Zelda's.
(B) Zelda's quiz average for the academic year was higher than Yolanda's.
(C) Yolanda's quiz average for the academic year was 3 points higher than Zelda's.
(D) Zelda's quiz average for the academic year equaled Yolanda's.
(E) If Zelda had scored 3 points higher on each quiz she took, then she would have had the same average for the academic year as Yolanda.

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19 Each of 2023 balls is placed in on of 3 bins. Which of the following is closest to the probability that each of the bins will contain an odd number of balls?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{4}$

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20 Cyrus the frog jumps 2 units in a direction, then 2 more in another direction. What is the probability that he lands less than 1 unit away from his starting position?

(I forgot answer choices)

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21 A lampshade is made in the form of the lateral surface of the frustum of a right circular cone. The height of the frustum is $3\sqrt{3}$ inches, its top diameter is 6 inches, and its bottom diameter is 12 inches. A bug is at the bottom of the lampshade and there is a glob of honey on the top edge of the lampshade at the spot farthest from the bug. The bug wants to crawl to the honey, but it must stay on the surface of the lampshade. What is the length in inches of its shortest path to the honey?

https://cdn.artofproblemsolving.com/attachments/b/4/23f9bc88ea057cb2676f2b8b373330b0f5df69.png

(A) $6+3\pi$ **(B)** $6+6\pi$ **(C)** $6\sqrt{3}$ **(D)** $6\sqrt{5}$ **(E)** $6\sqrt{3}+\pi$

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22 A real-valued function f has the property that for all real numbers a and b,

$$f(a+b)+f(a-b)=2f(a)f(b).$$

Which one of the following cannot be the value of $f(1)$? **(A)** 0 **(B)** 1 **(C)** -1 **(D)** 2 **(E)** -2

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23 When n standard six-sided dice are rolled, the product of the numbers rolled can be any of 936 possible values. What is n?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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24 Integers a, b, c, d satisfy the following: $abcd = 2^6 \cdot 3^9 \cdot 5^7$ lcm$(a, b) = 2^3 \cdot 3^2 \cdot 5^3$ lcm$(a, c) = 2^3 \cdot 3^3 \cdot 5^3$ lcm$(a, d) = 2^3 \cdot 3^3 \cdot 5^3$ lcm$(b, c) = 2^1 \cdot 3^3 \cdot 5^2$ lcm$(b, d) = 2^2 \cdot 3^3 \cdot 5^2$ lcm$(c, d) = 2^2 \cdot 3^3 \cdot 5^2$

Find gcd(a, b, c, d) **(A)** 30 **(B)** 45 **(C)** 3 **(D)** 15 **(E)** 6

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25 A regular pentagon with area $\sqrt{5}+1$ is printed on paper and cut out. The five vertices of the pentagon are folded into the center of the pentagon, creating a smaller pentagon. What is the area of the new pentagon?

(A) $4-\sqrt{5}$ **(B)** $\sqrt{5}-1$ **(C)** $8-3\sqrt{5}$ **(D)** $\frac{\sqrt{5}+1}{2}$ **(E)** $\frac{2+\sqrt{5}}{3}$

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