AMC 12/AHSME 1972

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1 The lengths in inches of the three sides of each of four triangles I, II, III, and IV are as follows:

I	$3,\ 4,$ and 5	III	$7,\ 24,$ and 25
II	$4,\ 7\frac{1}{2},$ and $8\frac{1}{2}$	IV	$3\frac{1}{2},\ 4\frac{1}{2},$ and $5\frac{1}{2}$.

Of these four given triangles, the only right triangles are

(A) I and II (B) I and III (C) I and IV
(D) I, II, and III (E) I, II, and IV

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2 If a dealer could get his goods for 8% less while keeping his selling price fixed, his profit, based on cost, would be increased to $(x+10)\%$ from his present profit of $x\%$, which is

(A) 12% **(B)** 15% **(C)** 30% **(D)** 50% **(E)** 75%

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3 If $x=\frac{1-i\sqrt{3}}{2}$ where $i=\sqrt{-1}$, then $\frac{1}{x^2-x}$ is equal to

(A) -2 **(B)** -1 **(C)** $1+i\sqrt{3}$ **(D)** 1 **(E)** 2

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4 The number of solutions to $\{1,\ 2\}\subseteq\ X\ \subseteq\ \{1,\ 2,\ 3,\ 4,\ 5\}$, where X is a subset of $\{1,\ 2,\ 3,\ 4,\ 5\}$ is

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** None of these

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5 From among $2^{1/2},\ 3^{1/3},\ 8^{1/8},\ 9^{1/9}$ those which have the greatest and the next to the greatest values, in that order, are

(A) $3^{1/3},\ 2^{1/2}$ (B) $3^{1/3},\ 8^{1/8}$ (C) $3^{1/3},\ 9^{1/9}$ (D) $8^{1/8},\ 9^{1/9}$
(E) None of these

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6 If $3^{2x}+9=10(3^x)$, then the value of (x^2+1) is

(A) 1 only **(B)** 5 only **(C)** 1 or 5 **(D)** 2 **(E)** 10

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7 If $yz:zx:xy=1:2:3$, then $\frac{x}{yz}:\frac{y}{zx}$ is equal to

(A) $3:2$ **(B)** $1:2$ **(C)** $1:4$ **(D)** $2:1$ **(E)** $4:1$

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8 If $|x-\log y| = x+\log y$ where x and $\log y$ are real, then

(A) $x=0$ **(B)** $y=1$ **(C)** $x=0$ and $y=1$

(D) $x(y-1)=0$ **(E)** None of these

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9 Ann and Sue bought identical boxes of stationery. Ann used hers to write 1-sheet letters and Sue used hers to write 3-sheet letters. Ann used all the envelopes and had 50 sheets of paper left, while Sue used all of the sheets of paper and had 50 envelopes left. The number of sheets of paper in each box was

(A) 150 (B) 125 (C) 120 (D) 100 (E) 80

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10 For x real, the inequality $1 \le |x-2| \le 7$ is equivalent to

(A) $x \le 1$ or $x \ge 3$ **(B)** $1 \le x \le 3$ **(C)** $-5 \le x \le 9$

(D) $-5 \le x \le 1$ or $3 \le x \le 9$ **(E)** $-6 \le x \le 1$ or $3 \le x \le 10$

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11 The value(s) of y for which the following pair of equations

$$x^2+y^2-16=0 \text{ and } x^2-3y+12=0$$

may have a real common solution, are

(A) 4 only **(B)** $-7,\ 4$ **(C)** $0,\ 4$ **(D)** no y **(E)** all y

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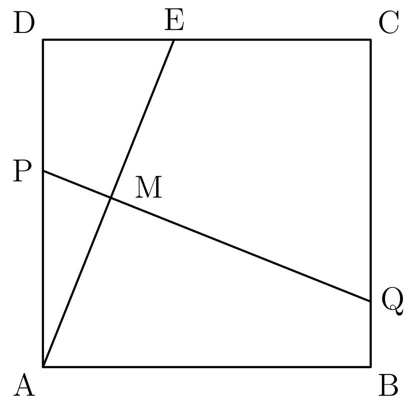
12 The number of cubic feet in the volume of a cube is the same as the number of square inches in its surface area. The length of the edge expressed as a number of feet is

(A) 6 **(B)** 864 **(C)** 1728 **(D)** 6×1728 **(E)** 2304

13



Inside square $ABCD$ (See figure) with sides of length 12 inches, segment AE is drawn where E is the point on DC which is 5 inches from D. The perpendicular bisector of AE is drawn and intersects AE, AD, and BC at points M, P, and Q respectively. The ratio of segment PM to MQ is

(A) $5:12$ **(B)** $5:13$ **(C)** $5:19$ **(D)** $1:4$ **(E)** $5:21$

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14 A triangle has angles of 30° and 45°. If the side opposite the 45° angle has length 8, then the side opposite the 30° angle has length

(A) 4 **(B)** $4\sqrt{2}$ **(C)** $4\sqrt{3}$ **(D)** $4\sqrt{6}$ **(E)** 6

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15 A contractor estimated that one of his two bricklayers would take 9 hours to build a certain wall and the other 10 hours. However, he knew from experience that when they worked together, their combined output fell by 10 bricks per hour. Being in a hurry, he put both men on the job and found that it took exactly 5 hours to build the wall. The number of bricks in the wall was

(A) 500 **(B)** 550 **(C)** 900 **(D)** 950 **(E)** 960

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16 There are two positive numbers that may be inserted between 3 and 9 such that the first three are in geometric progression while the last three are in arithmetic progression. The sum of those two positive numbers is

(A) $13\frac{1}{2}$ **(B)** $11\frac{1}{4}$ **(C)** $10\frac{1}{2}$ **(D)** 10 **(E)** $9\frac{1}{2}$

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17 A piece of string is cut in two at a point selected at random. The probability that the longer piece is at least x times as large as the shorter piece is

(A) $\frac{1}{2}$ **(B)** $\frac{2}{x}$ **(C)** $\frac{1}{x+1}$ **(D)** $\frac{1}{x}$ **(E)** $\frac{2}{x+1}$

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18 Let $ABCD$ be a trapezoid with the measure of base AB twice that of base DC, and let E be the point of intersection of the diagonals. If the measure of diagonal AC is 11, then that of segment EC is equal to

(A) $3\frac{2}{3}$ **(B)** $3\frac{3}{4}$ **(C)** 4 **(D)** $3\frac{1}{2}$ **(E)** 3

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19 The sum of the first n terms of the sequence

$$1,\ (1+2),\ (1+2+2^2),\ \dots\ (1+2+2^2+\cdots+2^{n-1})$$

in terms of n is

(A) 2^n **(B)** 2^n-n **(C)** $2^{n+1}-n$ **(D)** $2^{n+1}-n-2$ **(E)** $n\cdot 2^n$

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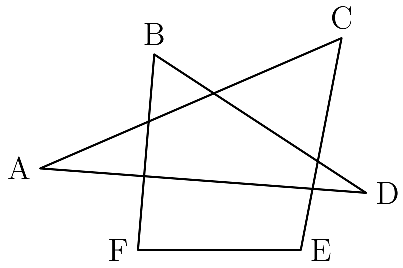
20 If $\tan x = \dfrac{2ab}{a^2-b^2}$ where $a>b>0$ and $0^\circ < x < 90^\circ$, then $\sin x$ is equal to

(A) $\frac{a}{b}$ **(B)** $\frac{b}{a}$ **(C)** $\frac{\sqrt{a^2-b^2}}{2a}$ **(D)** $\frac{\sqrt{a^2-b^2}}{2ab}$ **(E)** $\frac{2ab}{a^2+b^2}$

21



If the sum of the measures in degrees of angles $A,\ B,\ C,\ D,\ E$ and F in the figure above is $90n$, then n is equal to

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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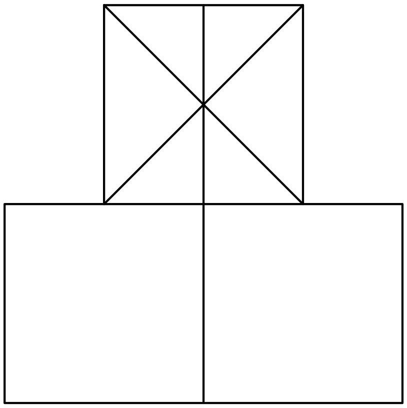
22 If $a \pm bi\ (b \neq 0)$ are imaginary roots of the equation $x^3 + qx + r = 0$ where $a,\ b,\ q,$ and r are real numbers, then q in terms of a and b is

(A) a^2+b^2 **(B)** $2a^2-b^2$ **(C)** b^2-a^2 **(D)** b^2-2a^2 **(E)** b^2-3a^2

23



The radius of the smallest circle containing the symmetric figure composed of the 3 unit squares shown above is

(A) $\sqrt{2}$ **(B)** $\sqrt{1.25}$ **(C)** 1.25 **(D)** $\frac{5\sqrt{17}}{16}$ **(E)** None of these

24 A man walked a certain distance at a constant rate. If he had gone $\frac{1}{2}$ mile per hour faster, he would have walked the distance in four-fifths of the time; if he had gone $\frac{1}{2}$ mile per hour slower, he would have been $2\frac{1}{2}$ hours longer on the road. The distance in miles he walked was

(A) $13\frac{1}{2}$ **(B)** 15 **(C)** $17\frac{1}{2}$ **(D)** 20 **(E)** 25

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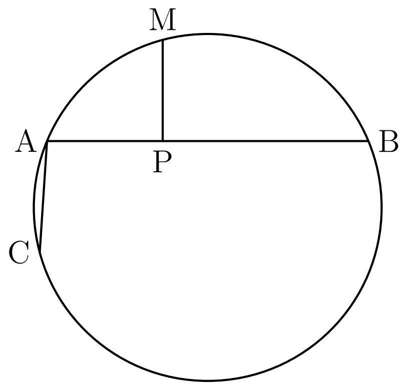
25 Inscribed in a circle is a quadrilateral having sides of lengths $25,\ 39,\ 52,$ and 60 taken consecutively. The diameter of this circle has length

(A) 62 **(B)** 63 **(C)** 65 **(D)** 66 **(E)** 69

26



In the circle above, M is the midpoint of arc CAB and segment MP is perpendicular to chord AB at P. If the measure of chord AC is x and that of segment AP is $(x+1)$, then segment PB has measure equal to

(A) $3x+2$ **(B)** $3x+1$ **(C)** $2x+3$ **(D)** $2x+2$ **(E)** $2x+1$

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27 If the area of $\triangle ABC$ is 64 square units and the geometric mean (mean proportional) between sides AB and AC is 12 inches, then $\sin A$ is equal to

(A) $\dfrac{\sqrt{3}}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{15}{17}$

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28 A circular disc with diameter D is placed on an 8×8 checkerboard with width D so that the centers coincide. The number of checkerboard squares which are completely covered by the disc is

(A) 48 **(B)** 44 **(C)** 40 **(D)** 36 **(E)** 32

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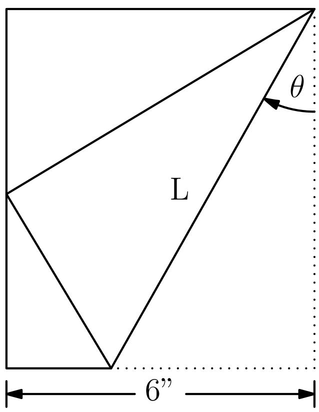
29 If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$, then $f\left(\frac{3x+x^3}{1+3x^2}\right)$ in terms of $f(x)$ is

(A) $-f(x)$ **(B)** $2f(x)$ **(C)** $3f(x)$
(D) $[f(x)]^2$ **(E)** $[f(x)]^3 - f(x)$

30



A rectangular piece of paper 6 inches wide is folded as in the diagram so that one corner touches the opposite side. The length in inches of the crease L in terms of angle θ is

(A) $3\sec^2\theta\csc\theta$ **(B)** $6\sin\theta\sec\theta$ **(C)** $3\sec\theta\csc\theta$ **(D)** $6\sec\theta\csc^2\theta$ **(E)** None of these

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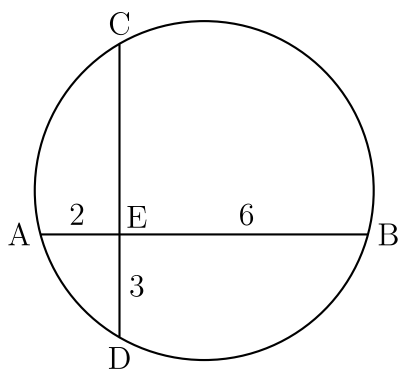
31 When the number 2^{1000} is divided by 13, the remainder in the division is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 7 **(E)** 11

32



Chords AB and CD in the circle above intersect at E and are perpendicular to each other. If segments AE, EB, and ED have measures 2, 3, and 6 respectively, then the length of the diameter of the circle is

(A) $4\sqrt{5}$ **(B)** $\sqrt{65}$ **(C)** $2\sqrt{17}$ **(D)** $3\sqrt{7}$ **(E)** $6\sqrt{2}$

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33 The minimum value of the quotient of a (base ten) number of three different nonzero digits divided by the sum of its digits is

(A) 9.7 **(B)** 10.1 **(C)** 10.5 **(D)** 10.9 **(E)** 20.5

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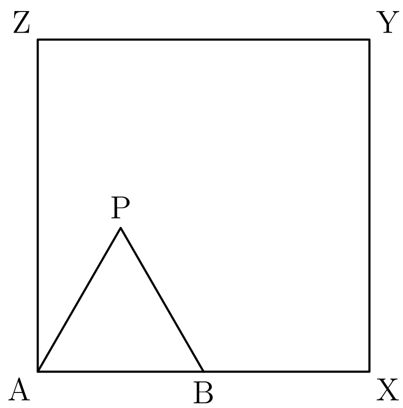
34 Three times Dick's age plus Tom's age equals twice Harry's age. Double the cube of Harry's age is equal to three times the cube of Dick's age added to the cube of Tom's age. Their respective ages are relatively prime to each other. The sum of the squares of their ages is

(A) 42 **(B)** 46 **(C)** 122 **(D)** 290 **(E)** 326

35



Equilateral triangle ABP (see figure) with side AB of length 2 inches is placed inside square $AXYZ$ with side of length 4 inches so that B is on side AX. The triangle is rotated clockwise about B, then P, and so on along the sides of the square until P returns to its original position. The length of the path in inches traversed by vertex P is equal to

(A) $20\pi/3$ **(B)** $32\pi/3$ **(C)** 12π **(D)** $40\pi/3$ **(E)** 15π

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