AMC 12/AHSME 1984

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1 $\frac{1000^2}{252^2-248^2}$ equals

(A) $62,500$ **(B)** 1000 **(C)** 500 **(D)** 250 **(E)** $\frac{1}{2}$

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2 If x, y and $y-\frac{1}{x}$ are not 0, then

$$\frac{x-\frac{1}{y}}{y-\frac{1}{x}}$$

equals

(A) 1 **(B)** $\frac{x}{y}$ **(C)** $\frac{y}{x}$ **(D)** $\frac{x}{y}-\frac{y}{x}$ **(E)** $xy-\frac{1}{xy}$

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3 Let n be the smallest nonprime integer greater than 1 with no prime factor less than 10. Then

A. $100 < n \leq 110$
B. $110 < n \leq 120$
C. $120 < n \leq 130$
D. $130 < n \leq 140$
E. $140 < n \leq 150$

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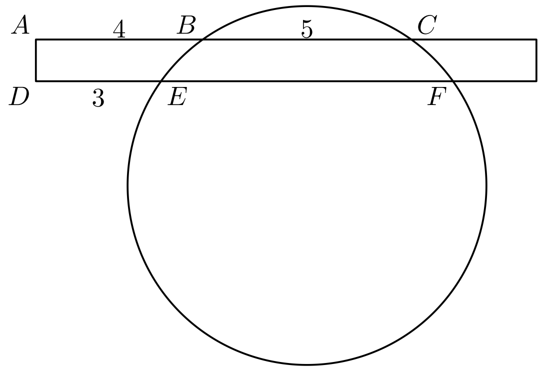
4 A rectangle intersects a circle as shown: $AB = 4$, $BC = 5$, and $DE = 3$. Then EF equals:



$(\mathbf{A})\ 6 \qquad (\mathbf{B})\ 7 \qquad (\mathbf{C})\ \frac{20}{3} \qquad (\mathbf{D})\ 8 \qquad (\mathbf{E})\ 9$

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5 The largest integer n for which $n^{200} < 5^{300}$ is

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

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6 In a certain school, there are three times as many boys as girls and nine times as many girls as teachers. Using the letters b, g, t to represent the number of boys, girls, and teachers, respectively, then the total number of boys, girls, and teachers can be represented by the expression

(A) $31b$ **(B)** $\frac{37b}{27}$ **(C)** $13g$ **(D)** $\frac{37g}{27}$ **(E)** $\frac{37t}{27}$

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7 When Dave walks to school, he averages 90 steps per minute, each of his steps 75cm long. It takes him 16 minutes to get to school. His brother, Jack, going to the same school by the same route, averages 100 steps per minute, but his steps are only 60 cm long. How long does it take Jack to get to school?

(A) $14\frac{2}{9}$ **(B)** 15 **(C)** 18 **(D)** 20 **(E)** $22\frac{2}{9}$

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8 Figure $ABCD$ is a trapezoid with $AB||DC, AB = 5, BC = 3\sqrt{2}, \measuredangle BCD = 45^\circ$, and $\measuredangle CDA = 60^\circ$. The length of DC is

(A) $7 + \frac{2}{3}\sqrt{3}$ **(B)** 8 **(C)** $9\frac{1}{2}$ **(D)** $8 + \sqrt{3}$ **(E)** $8 + 3\sqrt{3}$

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9 The number of digits in $4^{16}5^{25}$ (when written in the usual base 10 form) is

A. 31
B. 30
C. 29
D. 28
E. 27

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10 Four complex numbers lie at the vertices of a square in the complex plane. Three of the numbers are $1 + 2i, -2 + i$ and $-1 - 2i$. The fourth number is

A. $2 + i$
B. $2 - i$
C. $1 - 2i$
D. $-1 + 2i$
E. $-2 - i$

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11 A calculator has a key which replaces the displayed entry with its square, and another key which replaces the displayed entry with its reciprocal. Let y be the final result if one starts with an entry $x \neq 0$ and alternately squares and reciprocates n times each. Assuming the calculator is completely accurate (e.g., no roundoff or overflow), then y equals

A. $x^{((-2)^n)}$
B. x^{2n}

C. x^{-2n}
D. $x^{-(2^n)}$
E. $x^{((-1)^n 2n)}$

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12 If the sequence $\{a_n\}$ is defined by

$$\begin{aligned} a_1 &= 2, \\ a_{n+1} &= a_n + 2n \qquad (n \geq 1), \end{aligned}$$

then a_{100} equals

(A) 9900 **(B)** 9902 **(C)** 9904 **(D)** 10100 **(E)** 10102

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13 $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ equals

A. $\sqrt{2}+\sqrt{3}-\sqrt{5}$
B. $4-\sqrt{2}-\sqrt{3}$
C. $\sqrt{2}+\sqrt{3}+\sqrt{6}-5$
D. $\frac{1}{2}(\sqrt{2}+\sqrt{5}-\sqrt{3})$
E. $\frac{1}{3}(\sqrt{3}+\sqrt{5}-\sqrt{2})$

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14 The product of all real roots of the equation $x^{\log_{10} x} = 10$ is

A. 1
B. -1
C. 10
D. 10^{-1}
E. None of these

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15 If $\sin 2x \sin 3x = \cos 2x \cos 3x$, then one value for x is

A. 18°
B. 30°
C. 36°
D. 45°
E. 60°

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16 The function $f(x)$ satisfies $f(2+x) = f(2-x)$ for all real numbers x. If the equation $f(x) = 0$ has exactly four distinct real roots, then the sum of these roots is

A. 0
B. 2
C. 4

D. 6
E. 8

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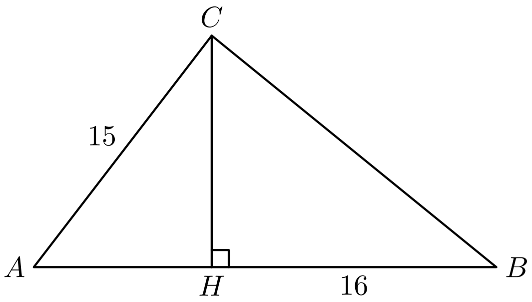
17 A right triangle ABC with hypotenuse AB has side $AC = 15$. Altitude CH divides AB into segments AH And HB, with $HB = 16$. The area of $\triangle ABC$ is:



(A) 120 **(B)** 144 **(C)** 150 **(D)** 216 **(E)** $144\sqrt{5}$

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18 A point (x, y) is to be chosen in the coordinate plane so that it is equally distant from the x-axis, the y-axis, and the line $x + y = 2$. Then x is

A. $\sqrt{2} - 1$
B. $\frac{1}{2}$
C. $2 - \sqrt{2}$
D. 1
E. Not uniquely determined

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19 A box contains 11 balls, numbered 1,2,3,....,11. If 6 balls are drawn simultaneously at random, what is the probability that the sum of the numbers on the balls drawn is odd?

A. $\frac{100}{231}$
B. $\frac{115}{231}$
C. $\frac{1}{2}$
D. $\frac{118}{231}$
E. $\frac{6}{11}$

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20 The number of distinct solutions of the equation $\big|x - |2x + 1|\big| = 3$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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21 The number of triples (a, b, c) of positive integers which satisfy the simultaneous equations

$$ab + bc = 44,$$
$$ac + bc = 23,$$

is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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22 Let a and c be fixed positive numbers. For each real number t let (x_t, y_t) be the vertex of the parabola $y = ax^2 + bx + c$. If the set of vertices (x_t, y_t) for all real values of t is graphed in the plane, the graph is

A. a straight line
B. a parabola
C. part, but not all, of a parabola
D. one branch of a hyperbola
E. None of these

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23 $\frac{\sin 10^\circ + \sin 20^\circ}{\cos 10^\circ + \cos 20^\circ}$ equals

A. $\tan 10^\circ + \tan 20^\circ$
B. $\tan 30^\circ$
C. $\frac{1}{2}(\tan 10^\circ + \tan 20^\circ)$
D. $\tan 15^\circ$
E. $\frac{1}{4}\tan 60^\circ$

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24 If a and b are positive real numbers and each of the equations

$$x^2 + ax + 2b = 0 \quad \text{and} \quad x^2 + 2bx + a = 0$$

has real roots, then the smallest possible value of $a + b$ is

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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25 The total area of all the faces of a rectangular solid is 22cm^2, and the total length of all its edges is 24cm. Then the length in cm of any one of its internal diagonal is

A. $\sqrt{11}$
B. $\sqrt{12}$
C. $\sqrt{13}$
D. $\sqrt{14}$
E. Not uniquely determined

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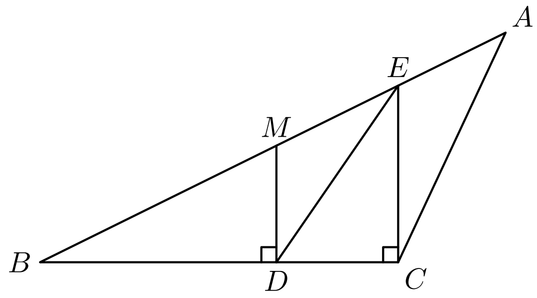
26 In the obtuse triangle ABC, $AM = MB, MD \perp BC, EC \perp BC$. If the area of $\triangle ABC$ is 24, then the area of $\triangle BED$ is



(A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** not uniquely determined

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27 In $\triangle ABC$, D is on AC and F is on BC. Also, $AB \perp AC, AF \perp BC$, and $BD = DC = FC = 1$. Find AC.

A. $\sqrt{2}$
B. $\sqrt{3}$
C. $\sqrt[3]{2}$
D. $\sqrt[3]{3}$
E. $\sqrt[4]{3}$

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28 The number of distinct pairs of integers (x, y) such that

$$0 < x < y \quad \text{and} \quad \sqrt{1984} = \sqrt{x} + \sqrt{y}$$

is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 7

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29 Find the largest value for $\frac{y}{x}$ for pairs of real numbers (x, y) which satisfy

$$(x-3)^2 + (y-3)^2 = 6.$$

(A) $3 + 2\sqrt{2}$ **(B)** $2 + \sqrt{3}$ **(C)** $3\sqrt{3}$ **(D)** 6 **(E)** $6 + 2\sqrt{3}$

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30 For any complex number $w = a + bi$, $|w|$ is defined to be the real number $\sqrt{a^2 + b^2}$. If $w = \cos 40^\circ + i \sin 40^\circ$, then

$$|w + 2w^2 + 3w^3 + \cdots + 9w^9|^{-1}$$

equals

(A) $\frac{1}{9} \sin 40^\circ$ **(B)** $\frac{2}{9} \sin 20^\circ$ **(C)** $\frac{1}{9} \cos 40^\circ$ **(D)** $\frac{1}{18} \cos 20^\circ$ **(E)** none of these

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