AMC 12/AHSME 1986

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1 $[x-(y-x)]-[(x-y)-x]=$

(A) $2y$ **(B)** $2x$ **(C)** $-2y$ **(D)** $-2x$ **(E)** 0

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2 If the line L in the xy-plane has half the slope and twice the y-intercept of the line $y=\frac{2}{3}x+4$, then an equation for L is:

(A) $y=\frac{1}{3}x+8$ **(B)** $y=\frac{4}{3}x+2$ **(C)** $y=\frac{1}{3}x+4$
(D) $y=\frac{4}{3}x+4$ **(E)** $y=\frac{1}{3}x+2$

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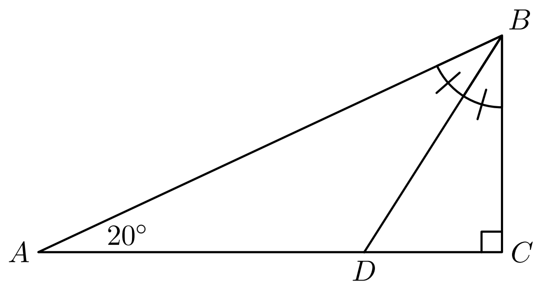
3 $\triangle ABC$ is a right angle at C and $\angle A=20^\circ$. If BD is the bisector of $\angle ABC$, then $\angle BDC=$



(A) 40° **(B)** 45° **(C)** 50° **(D)** 55° **(E)** 60°

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4 Let S be the statement

”If the sum of the digits of the whole number n is divisible by 6, then n is divisible by 6.”

A value of n which shows S to be false is

(A) 30 **(B)** 33 **(C)** 40 **(D)** 42 **(E)** None of these

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5 Simplify $\left(\sqrt[6]{27}-\sqrt{6\frac{3}{4}}\right)^2$

(A) $\frac{3}{4}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{3\sqrt{3}}{4}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{3\sqrt{3}}{2}$

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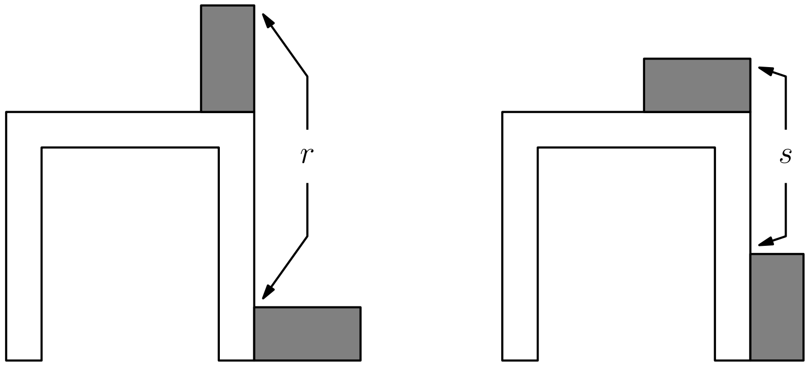
6 Using a table of a certain height, two identical blocks of wood are placed as shown in Figure 1. Length r is found to be 32 inches. After rearranging the blocks as in Figure 2, length s is found to be 28 inches. How high is the table?



(A) 28 inches **(B)** 29 inches **(C)** 30 inches **(D)** 31 inches **(E)** 32 inches

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7 The sum of the greatest integer less than or equal to x and the least integer greater than or equal to x is 5. The solution set for x is

(A) $\left\{\frac{5}{2}\right\}$ **(B)** $\{x \mid 2 \le x \le 3\}$ **(C)** $\{x \mid 2 \le x < 3\}$
(D) $\left\{x \mid 2 < x \le 3\right\}$ **(E)** $\left\{x \mid 2 < x < 3\right\}$

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8 The population of the United States in 1980 was $226,504,825$. The area of the country is $3,615,122$ square miles. The are $(5280)^2$ square feet in one square mile. Which number below best approximates the average number of square feet per person?

(A) $5,000$ **(B)** $10,000$ **(C)** $50,000$ **(D)** $100,000$ **(E)** $500,000$

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9 The product

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\dots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$$

equals

(A) $\frac{5}{12}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{11}{20}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{7}{10}$

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10 The 120 permutations of the AHSME are arranged in dictionary order as if each were an ordinary five-letter word. The last letter of the 85th word in this list is:

(A) A **(B)** H **(C)** S **(D)** M **(E)** E

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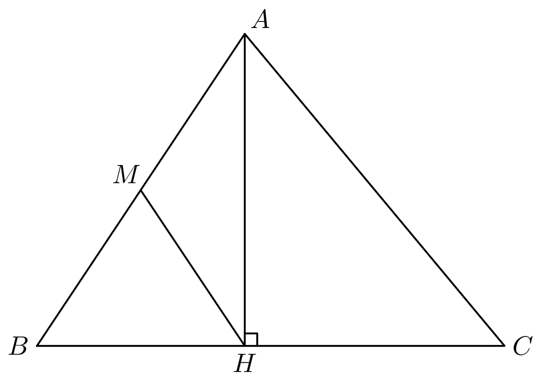
11 In $\triangle ABC$, $AB = 13$, $BC = 14$ and $CA = 15$. Also, M is the midpoint of side AB and H is the foot of the altitude from A to BC. The length of HM is



(A) 6 **(B)** 6.5 **(C)** 7 **(D)** 7.5 **(E)** 8

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12 John scores 93 on this year's AHSME. Had the old scoring system still been in effect, he would score only 84 for the same answers. How many questions does he leave unanswered? (In the new scoring system one receives 5 points for correct answers, 0 points for wrong answers, and 2 points for unanswered questions. In the old system, one started with 30 points, received 4 more for each correct answer, lost one point for each wrong answer, and neither gained nor lost points for unanswered questions. There are 30 questions in the 1986 AHSME.)

(A) 6 **(B)** 9 **(C)** 11 **(D)** 14 **(E)** Not uniquely determined

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13 A parabola $y = ax^2 + bx + c$ has vertex $(4, 2)$. If $(2, 0)$ is on the parabola, then abc equals

(A) -12 **(B)** -6 **(C)** 0 **(D)** 6 **(E)** 12

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14 Suppose hops, skips and jumps are specific units of length. If b hops equals c skips, d jumps equals e hops, and $f$jumps equals g meters, then one meter equals how many skips?

(A) $\frac{bdg}{cef}$ **(B)** $\frac{cdf}{beg}$ **(C)** $\frac{cdg}{bef}$ **(D)** $\frac{cef}{bdg}$ **(E)** $\frac{ceg}{bdf}$

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15 A student attempted to compute the average A of x, y and z by computing the average of x and y, and then computing the average of the result and z. Whenever $x < y < z$, the student's final result is

(A) correct **(B)** always less than A **(C)** always greater than A **(D)** sometimes less than A and sometimes equ
(E) sometimes greater than A and sometimes equal to A

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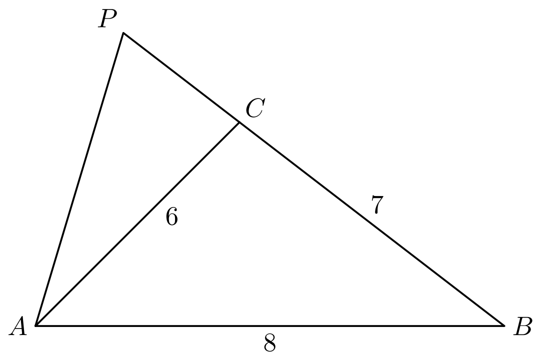
16 In $\triangle ABC$, $AB = 8$, $BC = 7$, $CA = 6$ and side BC is extended, as shown in the figure, to a point P so that $\triangle PAB$ is similar to $\triangle PCA$. The length of PC is



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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17 A drawer in a darkened room contains 100 red socks, 80 green socks, 60 blue socks and 40 black socks. A youngster selects socks one at a time from the drawer but is unable to see the color of the socks drawn. What is the smallest number of socks that must be selected to guarantee that the selection contains at least 10 pairs? (A pair of socks is two socks of the same color. No sock may be counted in more than one pair.)

(A) 21 **(B)** 23 **(C)** 24 **(D)** 30 **(E)** 50

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18 A plane intersects a right circular cylinder of radius 1 forming an ellipse. If the major axis of the ellipse of 50% longer than the minor axis, the length of the major axis is

(A) 1 **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{9}{4}$ **(E)** 3

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19 A park is in the shape of a regular hexagon 2 km on a side. Starting at a corner, Alice walks along the perimeter of the park for a distance of 5 km. How many kilometers is she from her starting point?

(A) $\sqrt{13}$ **(B)** $\sqrt{14}$ **(C)** $\sqrt{15}$ **(D)** $\sqrt{16}$ **(E)** $\sqrt{17}$

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20 Suppose x and y are inversely proportional and positive. If x increases by $p\%$, then y decreases by

(A) $p\%$ **(B)** $\frac{p}{1+p}\%$ **(C)** $\frac{100}{p}\%$ **(D)** $\frac{p}{100+p}\%$ **(E)** $\frac{100p}{100+p}\%$

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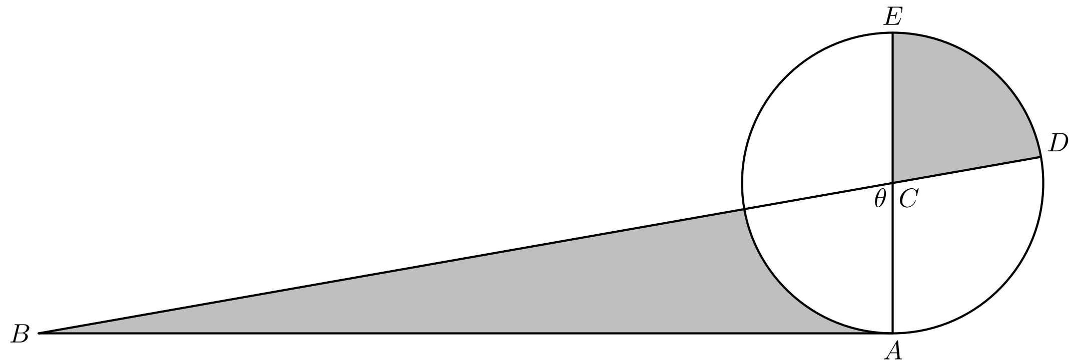
21 In the configuration below, θ is measured in radians, C is the center of the circle, BCD and ACE are line segments and AB is tangent to the circle at A.



A necessary and sufficient condition for the equality of the two shaded areas, given $0 < \theta < \frac{\pi}{2}$, is

(A) $\tan \theta = \theta$ **(B)** $\tan \theta = 2\theta$ **(C)** $\tan \theta = 4\theta$ **(D)** $\tan 2\theta = \theta$
(E) $\tan \frac{\theta}{2} = \theta$

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22 Six distinct integers are picked at random from $\{1, 2, 3, \ldots, 10\}$. What is the probability that, among those selected, the second smallest is 3?

(A) $\frac{1}{60}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** none of these

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23 Let

$$N = 69^5 + 5 \cdot 69^4 + 10 \cdot 69^3 + 10 \cdot 69^2 + 5 \cdot 69 + 1.$$

How many positive integers are factors of N?

(A) 3 **(B)** 5 **(C)** 69 **(D)** 125 **(E)** 216

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24 Let $p(x) = x^2 + bx + c$, where b and c are integers. If $p(x)$ is a factor of both

$$x^4 + 6x^2 + 25 \quad \text{and} \quad 3x^4 + 4x^2 + 28x + 5,$$

what is $p(1)$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

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25 If $\lfloor x \rfloor$ is the greatest integer less than or equal to x, then

$$\sum_{N=1}^{1024} \lfloor \log_2 N \rfloor =$$

(A) 8192 **(B)** 8204 **(C)** 9218 **(D)** $\lfloor \log_2(1024!) \rfloor$ **(E)** none of these

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26 It is desired to construct a right triangle in the coordinate plane so that its legs are parallel to the x and y axes and so that the medians to the midpoints of the legs lie on the lines $y = 3x + 1$ and $y = mx + 2$. The number of different constants m for which such a triangle exists is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

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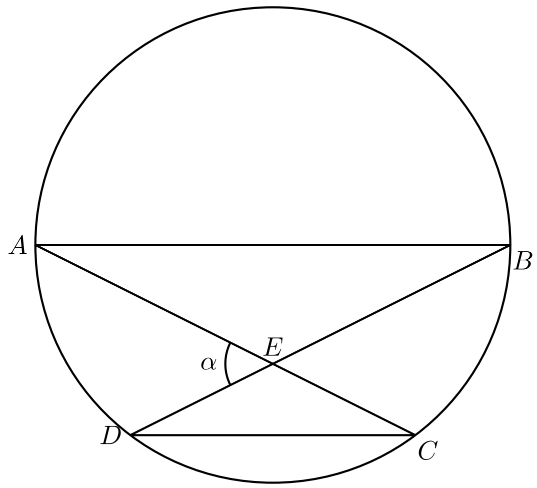
27 In the adjoining figure, AB is a diameter of the circle, CD is a chord parallel to AB, and AC intersects BD at E, with $\angle AED = \alpha$. The ratio of the area of $\triangle CDE$ to that of $\triangle ABE$ is



(A) $\cos\ \alpha$ **(B)** $\sin\ \alpha$ **(C)** $\cos^2 \alpha$ **(D)** $\sin^2 \alpha$ **(E)** $1 - \sin\ \alpha$

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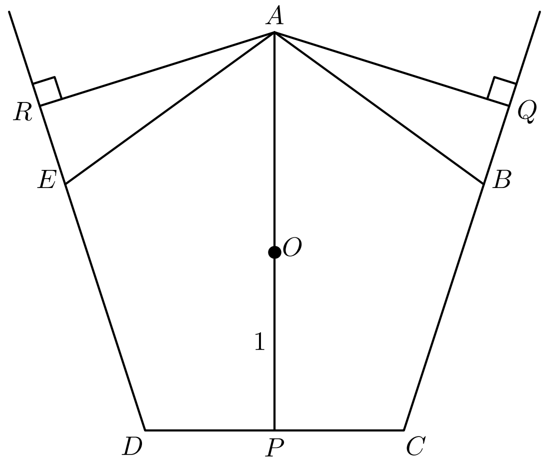
28 $ABCDE$ is a regular pentagon. AP, AQ and AR are the perpendiculars dropped from A onto CD, CB extended and DE extended, respectively. Let O be the center of the pentagon. If $OP = 1$, then $AO + AQ + AR$ equals



(A) 3 **(B)** $1+\sqrt{5}$ **(C)** 4 **(D)** $2+\sqrt{5}$ **(E)** 5

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29 Two of the altitudes of the scalene triangle ABC have length 4 and 12. If the length of the third altitude is also an integer, what is the biggest it can be?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** none of these

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30 The number of real solutions (x, y, z, w) of the simultaneous equations

$$2y = x + \frac{17}{x}, \quad 2z = y + \frac{17}{y}, \quad 2w = z + \frac{17}{z}, \quad 2x = w + \frac{17}{w}$$

is

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16
