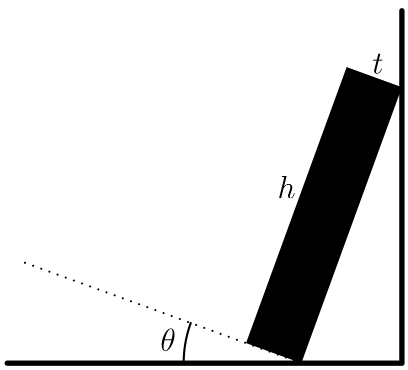
Syllabus 2023–2024

1. Static Equilibria, Stress, and Strain

This class reviews balancing forces and torques, the introduces the concepts of stress, strain, and Young's modulus.

Example Problem: A domino of height h, thickness t, and width w tilts at an angle θ, with its top touching a wall and its bottom touching a floor. If the wall is frictionless, find the minimum static coefficient of friction with the floor so that the domino does not slip.



Example Problem: The main cable of the Golden Gate Bridge is about $2300\ \mathrm{m}$ long when unstretched. It is made of steel with density $8\ \mathrm{g\ cm^{-3}}$ and has a Young's modulus of about $200\ \mathrm{GPa}$. Estimate the amount that the cable stretches when hung under its own weight.

2. Newton's Laws

Here, we'll build on your understanding of basic mechanics by applying Newton's laws to a variety of situations, familiar and unfamiliar.

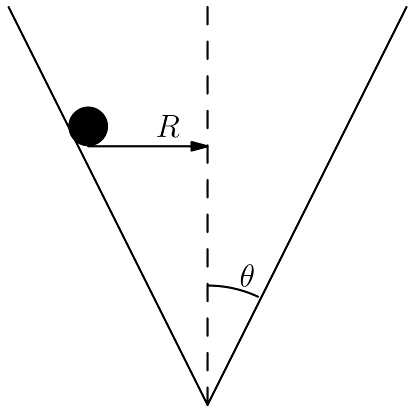
Example Problem: An airplane of mass m flies at speed v through air of density ρ. As viewed in the airplane's rest frame, air crossing the airplane's wings is deflected downward at an angle θ. Find the volume of air per second which the plane must deflect downward in order to fly at constant altitude. Building on this result, explain why planes with very large wings fly slowly and use less fuel per mile flown than planes with smaller wings.

Example Problem: The breaking strength of a human tendon is about $100\ \mathrm{MPa}$. Jumping from a building, a person lands on the balls of their feet and decelerates down a few inches until the heel of their foot hits the ground. Estimate the maximum height a person could jump from in this manner without breaking their Achilles tendon. State the assumptions that go into your model and briefly comment on how realistic your results are, and places your model might fail.

3. Rotational Motion

Rolling balls and cylinders with and without slipping form the first part. Then we discuss rotating reference frames and inertial forces.

Example Problem: A small marble rolls on the surface of a an upward-facing cone of angle θ. Find the period of the marble's orbit as a function of $r,$ the distance of the marble from the cone's axis.



Example Problem: A planet orbits its star in a circle (ignore any motion of the star). Seen in a rotating reference frame, the planet experiences a centrifugal force. Write this force as the negative of the gradient of some potential energy, called the effective potential. Find the effective potential in terms of the angular frequency of the planet's orbit $\omega,$ the planet's mass m, and distance r from the star. Explain whether the same potential will or will not apply to non-circular orbits as well. Then find the effective potential replacing ω with L, the planet's angular momentum, and answer the same question for this form.

4. Orbital Mechanics: Conservation Laws

Building on results from the first three classes, this class focuses on those problems in orbital mechanics best approached using energy and angular momentum. Several special results on energy apply specifically to orbits.

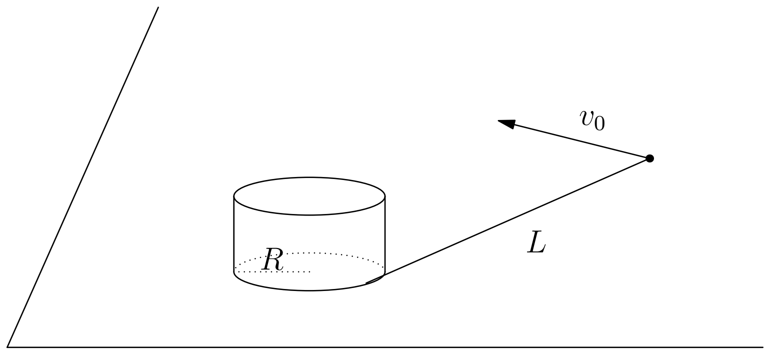
Example Problem: Suppose the moon suddenly stopped orbiting Earth. Find the time it would take for the Earth and Moon to collide, and find their relative velocity at that time. You may neglect the mass and radius of the moon compared to Earth, and ignore the possibility of the moon breaking apart before reaching Earth.

Example Problem: A rocket is launched from a pole of the Earth with the speed that would be necessary for a near-Earth orbit (but due to the angle of its launch, it doesn't go into this orbit). The rocket lands at the equator. Find the semi-major axis of the rocket's orbit. What is the maximal height of the rocket's orbit in terms of Earth's radius? What is the rocket's time of flight?

5. Friction

This class gives additional practice in mechanics by discussing friction in depth, including fluid viscosity and the origins of friction.

Example Problem: A round vertical cylinder of radius R is fixed on a horizontal plane. An inextensible thread of length L is attached at the cylinder side near the bottom. Initially the thread is tangent to the side. A small puck (of negligible size) is attached to the other end of the thread. The puck is given an initial velocity v_0 perpendicular to the thread, so the puck starts sliding on the plane. How long will the puck motion last if there is a coefficient of kinetic friction μ between the puck and the plane?



Example Problem: Two small disks with smooth lateral sides lie on a horizontal plane with a coefficient of kinetic friction μ. Initially, the first disk was at rest and the second one collided with it at a velocity $\vec{v}$. Determine the distance between the disks when they stop moving, provided the first disk has traveled the distance x_1. Assume the collision to be elastic but not necessarily central. What is the maximum and minimum finite distance between the disks for a given absolute value of velocity v and the coefficient of kinetic friction μ? Neglect the disk size. The free fall acceleration is g.

6. Approximation Tools and Error Analysis

Although we use approximations throughout the course, this class systematically introduces them and gives guidance on when and why to use them. They we discuss the meaning of error in physical measurements and introduce propagation of error.

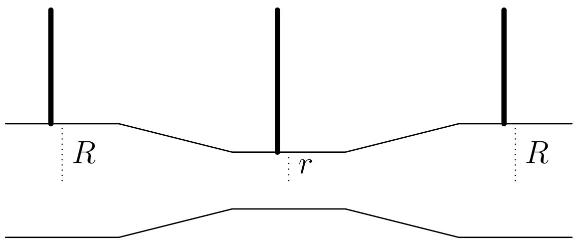
Example Problem: We often approximate the restoring torque on a pendulum using the approximation $\sin\theta \approx \theta$. Does this result in an overestimate or underestimate of the pendulum's period, or does it depend on the amplitude? Consider the approximation $\sin\theta \approx c\cdot\theta$ for some c not necessarily 1. Define a criterion for choosing c which you expect to lead to an improvement over $c = 1$ for estimating the period of the pendulum. Your criterion should lead to c being a function of $\theta_{\max}$. Find c for $\theta_{\max} = 45^\circ$ and evaluate the period of the pendulum. For a simple pendulum with a string length of $l = 1\ \mathrm{m}$ and $\theta_{\max} = 45^\circ$, the period, using $g = 10\ \mathrm{m\,s^{-2}}$, is $T \approx 2.066\ \mathrm{s}$. Was your new approximation an improvement over choosing $c = 1$?

Example Problem: You measure the parameters of the pendulum described in the previous problem, finding $l = 1\ \mathrm{m} \pm .2\ \mathrm{cm}, g = 10.0 \pm 0.1\ \mathrm{m\,s^{-2}}, \theta_{\max} = 45^\circ \pm 2^\circ$. Which of these measurements contributes the greatest uncertainty in your calculation of the period of the pendulum?

7. Fluid Dynamics

This class cover fluids in motion, including the continuity equation and energy conservation leading to Bernoulli's equation.

Example Problem: A horizontal tube has a narrow central section of radius r surrounded by two wider sections each of radius R. Three small vertical tubes extend from the top of the horizontal tube, one in each section. Fluid is pushed through the horizontal tube from left to right by a constant pressure difference ΔP from the left to the right end of the tube. Qualitatively describe the height that fluid will rise in each of the small vertical tubes assuming no frictional losses in the flow, then again assuming there are frictional losses.



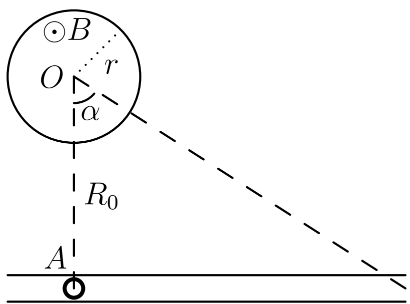
Example Problem: Suppose a soap bubble has radius R and surface tension σ. A small tube of radius $r \ll R$ is inserted into the soap bubble without breaking it, so that air begins rushing out of the soap bubble through the tube. The tube has negligible volume compared to the bubble. Find the radius of the bubble as a function of time. You may think of the air as an ideal incompressible fluid of density ρ.

8. Maxwell's Equations

This class summarizes all the fundamental laws of electromagnetism and introduces the displacement current.

Example Problem: Suppose we draw an Amperian loop in the shape of a circle between the plates of a parallel-plate capacitor, with the circle parallel to the planes of the plates. Because no charge moves through the circle, the current piercing through the Amperian loop is zero. Does this imply that the line integral of the magnetic field around the loop is zero? Why or why not?

Example Problem: A long solenoid of radius r produces a uniform magnetic field B_0 along its axis O. A straight tube AM made of a dielectric is fixed in a plane perpendicular to the axis at a distance R_0 from it. The angle AOM equals $\alpha = \pi/3$. The tube is much shorter than the solenoid. A small sphere of mass m and carrying positive charge q is placed inside the tube. Determine the sphere velocity at the moment of departure from the tube. Do this first for the case where the magnetic field quickly vanishes, so the sphere travels a distance much less than R_0 while the field dies away, and then for the case where the magnetic field decreases at a constant rate $\mathrm{d}B/\mathrm{d}t = -k < 0$ during the time of the motion of the sphere inside the tube. The friction and electromagnetic forces exerted by the tube on the sphere are negligible.

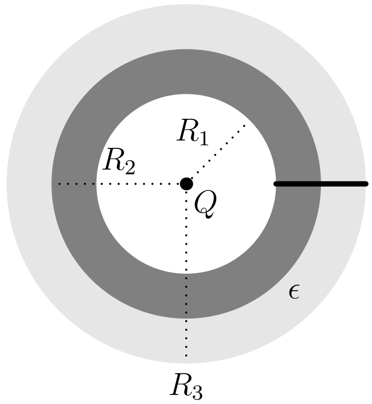


9. Electromagnetic Fields in Matter

This class studies dielectric materials, important in construction of capacitors, then introduces diamagnetism, ferromagnetism, and paramagnetism.

Example Problem: A solenoid consists of N loops of coil wrapped around a magnetic core of radius r. The core has magnetic permeability μ. The length of the solenoid is l. A current I is run through the solenoid. The core is pulled half way out of the solenoid. What force is required to hold the core in place?

Example Problem: A small sphere carrying charge Q is located at the center of a fixed uncharged conducting hollow sphere with outer and inner radii R_1 and R_2 $(R_2 < R_1)$. The sphere is enclosed by a concentric dielectric layer of permittivity ϵ and outer radius R_3. What is the minimum work required to move the small sphere to a distance much greater than R_3? You can assume there is a narrow channel inside the conductor and dielectric through which the small sphere can move.

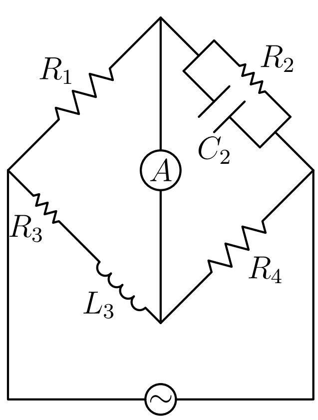


10. AC Circuits

Using complex impedances, this class analyzes circuits including resistors, capacitor, and inductors driven by a sinusoidal signal.

Example Problem: A coil of inductance 88.3 mH and unknown resistance and a 937 nF capacitor are connected in series with an oscillator of frequency 941 Hz. The phase angle ϕ between the applied emf and current is $75°$. Find the resistance of the coil.

Example Problem: A Wheatstone bridge circuit is used to determine the capacitance C_2 and leakage resistance R_2 of a certain capacitor. The bridge is balanced when a harmonic alternating voltage is applied. It turns out that the balance persists even under variations of the voltage frequency. Determine C_2 and R_2 in terms of $R_1, R_3, R_4,$ and L_3.



11. Theory of Waves

This class looks at principles that apply to all waves: Huygen's Principle, Fermat's Principle, and diffraction.

Example Problem: In 1838, Samuel Birley Rowbotham placed a series of poles of known height placed in the Old Bedford River in England. Rowbotham found that light rays traveled at a constant distance above the surface of the lake over the course of six miles, and concluded that the Earth is flat. Qualitatively explain why light may travel a long distance across the surface of the Earth remaining at approximately the same elevation, when traveling in a straight line would result in gaining elevation due to the Earth curving away underneath the path of the light. Estimate the temperature gradient in air, in degrees K per meter, needed for light to follow a path with the same radius of curvature as Earth. You can take the index of refraction of air to be 1.0003 at one atmosphere and $300\ \mathrm{K}$, and to vary linearly with the density of air.

Example Problem: One end of a stick is dragged through water at a speed v that is greater than the speed u of water waves. Applying Huygens' construction to the water waves, show that a conical wavefront is set up and that its half-angle α is given by $\sin\alpha = u/v$.

12. Sound and Electromagnetic Waves

The physics of two special types of waves - sound and electromagnetic - gives additional context to the abstract study of waves from the previous class.

Example Problem: In class, we derived the equation for the speed of sound in terms of universal constants, the molecular mass of air, and the adiabatic index of air. We assumed that sound waves compress air adiabatically. Isaac Newton attempted the same calculation, but made the assumption that sound wave compress air isothermally. By what factor was Newton's calculation incorrect?

Example Problem: High-power lasers are used to compress gas plasmas by radiation pressure. The reflectivity of a plasma is unity if the electron density is high enough. A laser generating pulses of radiation of peak power $1.5\ \mathrm{GW}$ is focused onto $1.3\ \mathrm{mm}^2$ of high-electron-density plasma. Find the pressure exerted on the plasma.

13. Polarization, Doppler Effect

These two special topics in waves lead to applications in optics and astrophysics.

Example Problem: A flare rocket flies at a constant speed v and generates a sound at a constant frequency f_0. Take the speed of sound to be $330\ \mathrm{m/s}$. If the rocket is directly approaching a tuning device, what frequency will the tuning device register? What if the rocket's velocity makes an angle θ with the line from the rocket to the sensor? Sketch a plot of the frequency that would be recorded if the rocket were to fly in a circle.

Example Problem: Suppose that two linear polarizers are rotated relative to each other by an angle θ. Find the fraction of energy of the light that passes through both polarizers. Show that for n polarizers, each rotated by an angle θ/n, the fraction of light energy transmitted through all polarizers goes to one half as $n \to \infty$.

14. Statistical Physics

This lesson introduces the kinetic theory of gases, then gives a statistical view of entropy and the Boltzmann distribution.

Example Problem: A simple estimate for the height of an isothermal atmosphere is to equate the mean kinetic energy of an air molecules, $\frac{1}{2}mv^2$, with the mean gravitational potential energy, mgh. Although this method is correct to order of magnitude, the equipartition theorem might suggest it to be exact within the model of an isothermal atmosphere (of a single species), whereas in fact it is wrong by a substantial factor. Why?

Example Problem: When doubling the temperature of a gas, by what factor does the mean speed of gas molecules increase? By what factor does the pressure increase? If these factors are not the same, why does the pressure change by a different factor than the speed, when microscopically, pressure on a wall of a container corresponds to collisions of molecules against the side of the container, and doubling the speed of the molecules doesn't the momentum transfer in a collision?

15. Relativity and Lorentz Transformations

All the common results in special relativity are unified by studying the Lorentz transformations, including four-vectors and conservation laws.

Example Problem: A photon of energy E bounces back and forth in a stationary cavity of mass m. Assume $\frac{E}{c^2} \ll m$. A constant force F pushes the cavity to the right. The average acceleration of the cavity, over many periods of oscillation of the photon, is Ma. You can assume the velocity of the cavity remains very small compared to the speed of light. Find M.

Example Problem: Train A has length L. Train B moves past A (on a parallel track, facing the same direction) with relative speed $4c/5$. The length of B is such that A says that the fronts of the trains coincide at exactly the same time as the backs coincide. What is the time difference between the fronts coinciding and the back coinciding, as measured by B?

16. Quantum Mechanics: Probability, Uncertainty, Atoms and Nuclei

Using the Heisenberg uncertainty principle as its primary tool, this lesson looks at important results in the basic atomic theory.

Example Problem: The mass of an electron is m_e and its charge is $-e$. Consider the nucleus of a hydrogen atom to be infinitely massive and have charge e. Imagine the electron to be confined to a ball centered on the nucleus with radius r. Estimate the kinetic energy of the electron from the uncertainty principle. Find r that minimizes the sum of the electrostatic potential energy of the system and the kinetic energy of the electron. Compare your result to the Bohr radius.

Example Problem: Use dimensional analysis to estimate the speed of sound in diamond from fundamental constants and m_n, the mass of a carbon nucleus, and m_e, the mass of an electron. Is dimensional analysis alone enough to determine this speed? If not, use our equations for the binding energy and radius of atoms, combined with your understanding of the theory of waves, to conjecture the correct equation up to a constant. Compare your result to the true speed of sound in diamond, about 10^4 m/s.

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