

Junior Balkan MO 2015

www.artofproblemsolving.com/community/c100694

by Sayan, neverlose

– June 26th

- 1** Find all prime numbers a, b, c and positive integers k satisfying the equation

$$a^2 + b^2 + 16c^2 = 9k^2 + 1.$$

Proposed by Moldova

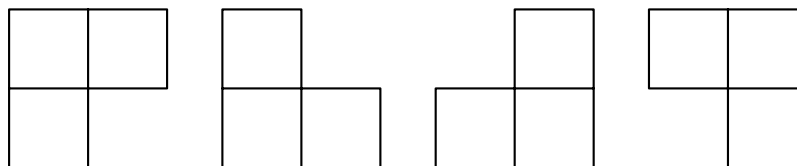
- 2** Let a, b, c be positive real numbers such that $a + b + c = 3$. Find the minimum value of the expression

$$A = \frac{2 - a^3}{a} + \frac{2 - b^3}{b} + \frac{2 - c^3}{c}.$$

- 3** Let ABC be an acute triangle. The lines l_1 and l_2 are perpendicular to AB at the points A and B , respectively. The perpendicular lines from the midpoint M of AB to the lines AC and BC intersect l_1 and l_2 at the points E and F , respectively. If D is the intersection point of the lines EF and MC , prove that

$$\angle ADB = \angle EMF.$$

- 4** An L-shape is one of the following four pieces, each consisting of three unit squares:



A 5×5 board, consisting of 25 unit squares, a positive integer $k \leq 25$ and an unlimited supply of L-shapes are given. Two players A and B, play the following game: starting with A they play alternatively mark a previously unmarked unit square until they marked a total of k unit squares.

We say that a placement of L-shapes on unmarked unit squares is called *good* if the L-shapes do not overlap and each of them covers exactly three unmarked unit squares of the board.

B wins if every *good* placement of L-shapes leaves uncovered at least three unmarked unit squares. Determine the minimum value of k for which B has a winning strategy.
