CentroAmerican 2015

www.artofproblemsolving.com/community/c100895

by fprosk

– Day 1

Problem 1 We wish to write n distinct real numbers $(n \geq 3)$ on the circumference of a circle in such a way that each number is equal to the product of its immediate neighbors to the left and right. Determine all of the values of n such that this is possible.

Problem 2 A sequence (a_n) of real numbers is defined by $a_0 = 1$, $a_1 = 2015$ and for all $n \geq 1$, we have

$$a_{n+1} = \frac{n-1}{n+1}a_n - \frac{n-2}{n^2+n}a_{n-1}.$$

Calculate the value of $\frac{a_1}{a_2} - \frac{a_2}{a_3} + \frac{a_3}{a_4} - \frac{a_4}{a_5} + \ldots + \frac{a_{2013}}{a_{2014}} - \frac{a_{2014}}{a_{2015}}$.

Problem 3 Let $ABCD$ be a cyclic quadrilateral with $AB < CD$, and let P be the point of intersection of the lines AD and BC.The circumcircle of the triangle PCD intersects the line AB at the points Q and R. Let S and T be the points where the tangents from P to the circumcircle of $ABCD$ touch that circle.

(a) Prove that $PQ = PR$.

(b) Prove that $QRST$ is a cyclic quadrilateral.

– Day 2

Problem 4 Anselmo and Bonifacio start a game where they alternatively substitute a number written on a board. In each turn, a player can substitute the written number by either the number of divisors of the written number or by the difference between the written number and the number of divisors it has. Anselmo is the first player to play, and whichever player is the first player to write the number 0 is the winner. Given that the initial number is 1036, determine which player has a winning strategy and describe that strategy.

Note: For example, the number of divisors of 14 is 4, since its divisors are 1, 2, 7, and 14.

Problem 5 Let ABC be a triangle such that $AC = 2AB$. Let D be the point of intersection of the angle bisector of the angle CAB with BC. Let F be the point of intersection of the line parallel to AB passing through C with the perpendicular line to AD passing through A. Prove that FD passes through the midpoint of AC.

Problem 6 39 students participated in a math competition. The exam consisted of 6 problems and each problem was worth 1 point for a correct solution and 0 points for an incorrect solution.

For any 3 students, there is at most 1 problem that was not solved by any of the three. Let B be the sum of all of the scores of the 39 students. Find the smallest possible value of B.