Junior Balkan MO 2019

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1   Find all prime numbers p for which there exist positive integers x, y, and z such that the number

$x^p + y^p + z^p - x - y - z$

is a product of exactly three distinct prime numbers.

2   Let a, b be two distinct real numbers and let c be a positive real numbers such that

$a^4 - 2019a = b^4 - 2019b = c.$

Prove that $-\sqrt{c} < ab < 0$.

3   Triangle ABC is such that $AB < AC$. The perpendicular bisector of side BC intersects lines AB and AC at points P and Q, respectively. Let H be the orthocentre of triangle ABC, and let M and N be the midpoints of segments BC and PQ, respectively. Prove that lines HM and AN meet on the circumcircle of ABC.

4   A 5×100 table is divided into 500 unit square cells, where n of them are coloured black and the rest are coloured white. Two unit square cells are called *adjacent* if they share a common side. Each of the unit square cells has at most two adjacent black unit square cells. Find the largest possible value of n.