Brazil Undergrad MO 2008

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3 Prove that there are real numbers $a_1, a_2, ..$ such that:

i) For all real numbers x, the serie $f(x) = \sum_{n=1}^{\infty} a_n x^n$ converge;
ii) f is a bijection of R to R;
iii) f'(x) ¿0;
iv) f(Q) = A, where Q is the set of rational numbers and A is the set of algebraic numbers.