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by randomusername

1 Let a, b be positive real numbers satisfying $2ab = a - b$. Denote for any positive integer k x_k and y_k to be the closest integer to ak and bk, respectively (if there are two closest integers, choose the larger one). Prove that any positive integer n appears in the sequence $(x_k)_{k\geq 1}$ if and only if it appears at least three times in the sequence $(y_k)_{k\geq 1}$.

2 Consider the closed polygonal discs P_1, P_2, P_3 with the property that for any three points $A \in P_1$, $B \in P_2$, $C \in P_3$, we have $[\triangle ABC] \leq 1$. (Here $[X]$ denotes the area of polygon X.)
(a) Prove that $\min\{[P_1], [P_2], [P_3]\} < 4$.
(b) Give an example of polygons P_1, P_2, P_3 with the above property such that $[P_1] > 4$ and $[P_2] > 4$.

3 Is it true that for integer $n \geq 2$, and given any non-negative reals ℓ_{ij}, $1 \leq i < j \leq n$, we can find a sequence $0 \leq a_1, a_2, \ldots, a_n$ such that for all $1 \leq i < j \leq n$ to have $|a_i - a_j| \geq \ell_{ij}$, yet still $\sum_{i=1}^{n} a_i \leq \sum_{1\leq i<j\leq n} \ell_{ij}$?