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by randomusername

- 1 Let J_A and J_B be the A -excenter and B -excenter of $\triangle ABC$. Consider a chord \overline{PQ} of circle ABC which is parallel to AB and intersects segments \overline{AC} and \overline{BC} . If lines AB and CP intersect at R , prove that

$$\angle J_A Q J_B + \angle J_A R J_B = 180^\circ.$$

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- 2 Denote by $E(n)$ the number of 1's in the binary representation of a positive integer n . Call n *interesting* if $E(n)$ divides n . Prove that
- (a) there cannot be five consecutive interesting numbers, and
 - (b) there are infinitely many positive integers n such that $n, n + 1$ and $n + 2$ are each interesting.

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- 3 Consider n events, each of which has probability $\frac{1}{2}$. We also know that the probability of any two both happening is $\frac{1}{4}$. Prove the following.
- (a) The probability that none of these events happen is at most $\frac{1}{n+1}$.
 - (b) We can reach equality in (a) for infinitely many n .
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