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by randomusername

1 We have n keys, each of them belonging to exactly one of n locked chests. Our goal is to decide which key opens which chest. In one try we may choose a key and a chest, and check whether the chest can be opened with the key. Find the minimal number $p(n)$ with the property that using $p(n)$ tries, we can surely discover which key belongs to which chest.

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2 Consider a triangle ABC, with the points A_1, A_2 on side BC, $B_1, B_2 \in \overline{AC}$, $C_1, C_2 \in \overline{AB}$ such that $AC_1 < AC_2, BA_1 < BA_2, CB_1 < CB_2$. Let the circles AB_1C_1 and AB_2C_2 meet at A and A^*. Similarly, let the circles BC_1A_1 and BC_2A_2 intersect at $B^* \neq B$, let CA_1B_1 and CA_2B_2 intersect at $C^* \neq C$. Prove that the lines AA^*, BB^*, CC^* are concurrent.

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3 For what positive integers n and k do there exits integers $a_1, a_2, \dots, a_n$ and $b_1, b_2, \dots, b_k$ such that the products a_ib_j $(1 \le i \le n, 1 \le j \le k)$ give pairwise different residues modulo nk?
