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by randomusername

1 Let n, k be arbitrary positive integers. We fill the entries of an $n \times k$ array with integers such that all the n rows contain the integers $1, 2, \ldots, k$ in some order. Add up the numbers in all k columns – let S be the largest of these sums. What is the minimal value of S?

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2 Find all positive integer pairs (a, b) for which the set of positive integers can be partitioned into sets H_1 and H_2 such that neither a nor b can be represented as the difference of two numbers in H_i for $i = 1, 2$.

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3 Find all functions $f : \mathbb{Z} \to \mathbb{Q}$ with the following properties: if $f(x) < c < f(y)$ for some rational c, then f takes on the value of c, and

$$f(x) + f(y) + f(z) = f(x)f(y)f(z)$$

whenever $x + y + z = 0$.
