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by randomusername

1 We have placed $n > 3$ cards around a circle, facing downwards. In one step we may perform the following operation with three consecutive cards. Calling the one on the center B, the two on the ends A and C, we put card C in the place of A, then move A and B to the places originally occupied by B and C, respectively. Meanwhile, we flip the cards A and B.

Using a number of these steps, is it possible to move each card to its original place, but facing upwards?

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2 Prove that if from any 2007 consecutive terms of an infinite arithmetic progression of integers starting with 2, one can choose a term relatively prime to all the 2006 other terms, then there is also a term amongst any 2008 consecutive terms relatively prime to the rest.

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3 Prove that any finite set H of lattice points on the plane has a subset K with the following properties:

-any vertical or horizontal line in the plane cuts K in at most 2 points,
-any point of $H \setminus K$ is contained by a segment with endpoints from K.
