

[www.artofproblemsolving.com/community/c103158](http://www.artofproblemsolving.com/community/c103158)

by randomusername

- 1 Let  $N > 1$  and let  $a_1, a_2, \dots, a_N$  be nonnegative reals with sum at most 500. Prove that there exist integers  $k \geq 1$  and  $1 = n_0 < n_1 < \dots < n_k = N$  such that

$$\sum_{i=1}^k n_i a_{n_{i-1}} < 2005.$$

- 
- 2 A and B play tennis. The player to first win at least four points and at least two more than the other player wins. We know that A gets a point each time with probability  $p \leq \frac{1}{2}$ , independent of the game so far. Prove that the probability that A wins is at most  $2p^2$ .
- 
- 3 We build a tower of  $2 \times 1$  dominoes in the following way. First, we place 55 dominoes on the table such that they cover a  $10 \times 11$  rectangle; this is the first story of the tower. We then build every new level with 55 dominoes above the exact same  $10 \times 11$  rectangle. The tower is called *stable* if for every non-lattice point of the  $10 \times 11$  rectangle, we can find a domino that has an inner point above it. How many stories is the lowest *stable* tower?
-