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by randomusername

1 Given is a triangle ABC, its circumcircle ω, and a circle k that touches ω from the outside, and also touches rays AB and AC in P and Q, respectively. Prove that the A-excenter of $\triangle ABC$ is the midpoint of $\overline{PQ}$.

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2 Find the smallest positive integer $n \neq 2004$ for which there exists a polynomial $f \in \mathbb{Z}[x]$ such that the equation $f(x) = 2004$ has at least one, and the equation $f(x) = n$ has at least 2004 different integer solutions.

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3 We have placed some red and blue points along a circle. The following operations are permitted:

(a) we may add a red point somewhere and switch the color of its neighbors,

(b) we may take off a red point from somewhere and switch the color of its neighbors (if there are at least 3 points on the circle and there is a red one too).

Initially, there are two blue points on the circle. Using a number of these operations, can we reach a state with exactly two red point?
