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by randomusername

1 Draw a circle k with diameter $\overline{EF}$, and let its tangent in E be e. Consider all possible pairs $A,B\in e$ for which $E\in \overline{AB}$ and $AE\cdot EB$ is a fixed constant. Define $(A_1,B_1)=(AF\cap k,BF\cap k)$. Prove that the segments $\overline{A_1B_1}$ all concur in one point.

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2 Prove that if a graph $\mathcal{G}$ on $n\ge 3$ vertices has a unique 3-coloring, then $\mathcal{G}$ has at least $2n-3$ edges.

(A graph is 3-colorable when there exists a coloring of its vertices with 3 colors such that no two vertices of the same color are connected by an edge. The graph can be 3-colored uniquely if there do not exist vertices u and v of the graph that are painted different colors in one 3-coloring, yet are colored the same in another.)

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3 Prove that the following inequality holds with the exception of finitely many positive integers n:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}gcd(i,j)>4n^2.$$
