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by randomusername

- 1** We have an acute-angled triangle which is not isosceles. We denote the orthocenter, the circumcenter and the incenter of it by H, O, I respectively. Prove that if a vertex of the triangle lies on the circle HOI , then there must be another vertex on this circle as well.

- 2** The Fibonacci sequence is defined as $f_1 = f_2 = 1, f_{n+2} = f_{n+1} + f_n$ ($n \in \mathbb{N}$). Suppose that a and b are positive integers such that $\frac{a}{b}$ lies between the two fractions $\frac{f_n}{f_{n-1}}$ and $\frac{f_{n+1}}{f_n}$. Show that $b \geq f_{n+1}$.

- 3** Prove that the edges of a complete graph with 3^n vertices can be partitioned into disjoint cycles of length 3.
