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by randomusername

1 $3n-1$ points are given in the plane, no three are collinear. Prove that one can select $2n$ of them whose convex hull is not a triangle.

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2 Let $k \geq 3$ be an integer. Prove that if $n > \binom{k}{3}$, then for any $3n$ pairwise different real numbers a_i, b_i, c_i $(1 \leq i \leq n)$, among the numbers a_i+b_i, a_i+c_i, b_i+c_i, one can find at least $k+1$ pairwise different numbers. Show that this is not always the case when $n = \binom{k}{3}$.

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3 In a square lattice let us take a lattice triangle that has the smallest area among all the lattice triangles similar to it. Prove that the circumcenter of this triangle is not a lattice point.
