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by randomusername

1 Paint the grid points of $L=\{0,1,\dots,n\}^2$ with red or green in such a way that every unit lattice square in L has exactly two red vertices. How many such colorings are possible?

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2 Let ABC be a non-equilateral triangle in the plane, and let T be a point different from its vertices. Define A_T, B_T and C_T as the points where lines AT, BT, and CT meet the circumcircle of ABC. Prove that there are exactly two points P and Q in the plane for which the triangles $A_PB_PC_P$ and $A_QB_QC_Q$ are equilateral. Prove furthermore that line PQ contains the circumcenter of $\triangle ABC$.

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3 Let $k\geq 0$ be an integer and suppose the integers $a_1,a_2,\dots,a_n$ give at least $2k$ different residues upon division by $(n+k)$. Show that there are some a_i whose sum is divisible by $n+k$.
