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by randomusername

1   For any positive integer m, denote by $d_i(m)$ the number of positive divisors of m that are congruent to i modulo 2. Prove that if n is a positive integer, then

$$\left| \sum_{k=1}^{n} \left(d_0(k) - d_1(k) \right) \right| \leq n.$$

2   Given a triangle on the plane, construct inside the triangle the point P for which the centroid of the triangle formed by the three projections of P onto the sides of the triangle happens to be P.

3   We are given more than 2^k integers, where $k \in \mathbb{N}$. Prove that we can choose $k+2$ of them such that if some of our selected numbers satisfy

$$x_1 + x_2 + \cdots + x_m = y_1 + y_2 + \cdots + y_m$$

where $x_1 < \cdots < x_m$ and $y_1 < \cdots < y_m$, then $x_i = y_i$ for any $1 \leq i \leq m$.