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by randomusername

- 1 Given in the plane is a lattice and a grid rectangle with sides parallel to the coordinate axes. We divide the rectangle into grid triangles with area $\frac{1}{2}$. Prove that the number of right angled triangles is at least twice as much as the shorter side of the rectangle.
(A grid polygon is a polygon such that both coordinates of each vertex is an integer.)

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- 2 Consider a polynomial in n variables with real coefficients. We know that if every variable is ± 1 , the value of the polynomial is positive, or negative if the number of -1 's is even, or odd, respectively. Prove that the degree of this polynomial is at least n .

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- 3 Points A, B, C, D are such that no three of them are collinear. Let $E = AB \cap CD$ and $F = BC \cap DA$. Let k_1, k_2 and k_3 denote the circles with diameter $\overline{AC}, \overline{BD}$ and \overline{EF} , respectively. Prove that either k_1, k_2, k_3 pass through one point, or no two of them intersect.
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