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by randomusername

**1**    Let $a$ and $b$ be positive integers. Prove that the numbers $an^2 + b$ and $a(n+1)^2 + b$ are both perfect squares only for finitely many integers $n$.

**2**    Triangle $ABC$ is not isosceles. The incircle of $\triangle ABC$ touches the sides $BC$, $CA$, $AB$ in the points $K$, $L$, $M$. The parallel with $LM$ through $B$ meets $KL$ at $D$, the parallel with $LM$ through $C$ meets $KM$ at $E$.

Prove that $DE$ passes through the midpoint of $\overline{LM}$.

**3**    Let $n$ be a fixed positive integer. Compute over $\mathbb{R}$ the minimum of the following polynomial:

$$f(x) = \sum_{t=0}^{2n} (2n + 1 - t)x^t.$$