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by randomusername

1 Let $p>2$ be a prime number and n a positive integer. Prove that pn^2 has at most one positive divisor d for which n^2+d is a square number.

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2 The incenter of $\triangle A_1A_2A_3$ is I, and the center of the A_i-excircle is J_i $(i=1,2,3)$. Let B_i be the intersection point of side $A_{i+1}A_{i+2}$ and the bisector of $\angle A_{i+1}IA_{i+2}$ $(A_{i+3}:=A_i\ \forall i)$. Prove that the three lines B_iJ_i are concurrent.

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3 We would like to give a present to one of 100 children. We do this by throwing a biased coin k times, after predetermining who wins in each possible outcome of this lottery.

Prove that we can choose the probability p of throwing heads, and the value of k such that, by distributing the 2^k different outcomes between the children in the right way, we can guarantee that each child has the same probability of winning.
