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by randomusername

1 In the plane, two intersecting lines a and b are given, along with a circle ω that has no common points with these lines. For any line $\ell \parallel b$, define $A = \ell \cap a$, and $\{B, C\} = \ell \cap \omega$ such that B is on segment AC . Construct the line ℓ such that the ratio $\frac{|BC|}{|AB|}$ is maximal.

2 For any positive integer n denote $S(n)$ the digital sum of n when represented in the decimal system. Find every positive integer M for which $S(Mk) = S(M)$ holds for all integers $1 \leq k \leq M$.

3 We play the following game in a Cartesian coordinate system in the plane. Given the input (x, y) , in one step, we may move to the point $(x, y \pm 2x)$ or to the point $(x \pm 2y, y)$. There is also an additional rule: it is not allowed to make two steps that lead back to the same point (i.e, to step backwards).

Prove that starting from the point $(1; \sqrt{2})$, we cannot return to it in finitely many steps.
