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by randomusername

1 Prove that if there exists a point P inside the convex quadrilateral $ABCD$ such that the triangles PAB, PBC, PCD, PDA have the same area, then one of the diagonals of $ABCD$ bisects the area of the quadrilateral.

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2 Set $T \subset \{1, 2, \ldots, n\}^3$ has the property that for any two triplets (a, b, c) and (x, y, z) in T, we have $a < b < c$, and also, we know that at most one of the equalities $a = x$, $b = y$, $c = z$ holds. Maximize $|T|$.

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3 Consider the convex lattice quadrilateral $PQRS$ whose diagonals intersect at E. Prove that if $\angle P + \angle Q < 180^\circ$, then the $\triangle PQE$ contains inside it or on one of its sides a lattice point other than P and Q.
