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by randomusername

1 Any two members of a club with $3n+1$ people plays ping-pong, tennis or chess with each other. Everyone has exactly n partners who plays ping-pong, n who play tennis and n who play chess.

Prove that we can choose three members of the club who play three different games amongst each other.

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2 Let $n > 2$ be a positive integer. Find the largest value h and the smallest value H for which

$$h < \frac{a_1}{a_1 + a_2} + \frac{a_2}{a_2 + a_3} + \cdots + \frac{a_n}{a_n + a_1} < H$$

holds for any positive reals $a_1, \ldots, a_n$.

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3 A and B plays the following game: they choose randomly k integers from $\{1, 2, \ldots, 100\}$; if their sum is even, A wins, else B wins. For what values of k does A and B have the same chance of winning?
