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by randomusername

1 We have triangulated a convex $(n+1)$-gon $P_0P_1 \ldots P_n$ (i.e., divided it into $n-1$ triangles with $n-2$ non-intersecting diagonals). Prove that the resulting triangles can be labelled with the numbers $1, 2, \ldots, n-1$ such that for any $i \in \{1, 2, \ldots, n-1\}$, P_i is a vertex of the triangle with label i.

2 For every $n \in \mathbb{N}$, define the *power sum* of n as follows. For every prime divisor p of n, consider the largest positive integer k for which $p^k \leq n$, and sum up all the p^k's. (For instance, the power sum of 100 is $2^6 + 5^2 = 89$.) Prove that the *power sum* of n is larger than n for infinitely many positive integers n.

3 We reflected each vertex of a triangle on the opposite side. Prove that the area of the triangle formed by these three reflection points is smaller than the area of the initial triangle multiplied by five.