Turkey Junior National Olympiad 2019

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by electrovector

1   Solve $2a^2 + 3a - 44 = 3p^n$ in positive integers where p is a prime.

2 $x, y, z \in \mathbb{R}^+$ and $x^5 + y^5 + z^5 = xy + yz + zx$. Prove that

$$3 \geq x^2y + y^2z + z^2x$$

3   In ABC triangle I is incenter and incircle of ABC tangents to BC, AC, AB at D, E, F, respectively. If AI intersects DE and DF at P and Q, prove that the circumcenter of DPQ triangle is the midpoint of BC.

4   There are 27 cardboard and 27 plastic boxes. There are balls of certain colors inside the boxes. It is known that any two boxes of the same kind do not have a ball with the same color. Boxes of different kind have at least one ball of the same color. At each step we select two boxes that have a ball of same color and switch this common color into any other color we wish. Find the smallest number n of moves required.