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by Xixas, randomusername

1 Show that if $a+b+c=0$ then $(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b})(\frac{b-c}{a}+\frac{c-a}{b}+\frac{a-b}{c})=9$.

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2 Suppose that $n \geq 8$ persons $P_1, P_2, \dots, P_n$ meet at a party. Assume that P_k knows $k+3$ persons for $k=1,2,\dots,n-6$. Further assume that each of $P_{n-5}, P_{n-4}, P_{n-3}$ knows $n-2$ persons, and each of P_{n-2}, P_{n-1}, P_n knows $n-1$ persons. Find all integers $n \geq 8$ for which this is possible.

(It is understood that "to know" is a symmetric nonreflexive relation: if P_i knows P_j then P_j knows P_i; to say that P_i knows p persons means: knows p persons other than herself/himself.)

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3 In a convex quadrilateral of area 1, the sum of the lengths of all sides and diagonals is not less than $4+\sqrt{8}$. Prove this.

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4 Solve the system of equations: $\begin{cases} x^4+y^2-xy^3-\dfrac{9}{8}x=0 \\ y^4+x^2-yx^3-\dfrac{9}{8}y=0 \end{cases}$

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5 We are given a certain number of identical sets of weights; each set consists of four different weights expressed by natural numbers (of weight units). Using these weights we are able to weigh out every integer mass up to 1985 (inclusive). How many ways are there to compose such a set of weight sets given that the joint mass of all weights is the least possible?

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6 Let P be a point inside a tetrahedron $ABCD$ and let S_A, S_B, S_C, S_D be the centroids (i.e. centers of gravity) of the tetrahedra $PBCD, PCDA, PDAB, PABC$. Show that the volume of the tetrahedron $S_AS_BS_CS_D$ equals $1/64$ the volume of $ABCD$.

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7 Find an upper bound for the ratio

$$\frac{x_1x_2+2x_2x_3+x_3x_4}{x_1^2+x_2^2+x_3^2+x_4^2}$$

over all quadruples of real numbers $(x_1,x_2,x_3,x_4) \neq (0,0,0,0)$.

Note. The smaller the bound, the better the solution.

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8 A convex n-gon $A_0A_1\dots A_{n-1}$ has been partitioned into $n-2$ triangles by certain diagonals not intersecting inside the n-gon. Prove that these triangles can be labeled $\triangle_1, \triangle_2, \dots, \triangle_{n-2}$ in such a way that A_i is a vertex of $\triangle_i$, for $i=1,2,\dots,n-2$. Find the number of all such labellings.

9 We are given a convex polygon. Show that one can find a point Q inside the polygon and three vertices A_1, A_2, A_3 (not necessarily consecutive) such that each ray A_iQ ($i = 1, 2, 3$) makes acute angles with the two sides emanating from A_i.