Lusophon Mathematical Olympiad 2019

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by parmenides51

–	Day 1
1	Find a way to write all the digits of 1 to 9 in a sequence and without repetition, so that the numbers determined by any two consecutive digits of the sequence are divisible by 7 or 13.
2	Prove that for every n nonzero integer , there are infinite triples of nonzero integers a, b and c that satisfy the conditions: 1. $a + b + c = n$ 2. $ax^2 + bx + c = 0$ has rational roots.
3	Let ABC be a triangle with $AC \neq BC$. In triangle ABC, let G be the centroid, I the incenter and O Its circumcenter. Prove that IG is parallel to AB if, and only if, CI is perpendicular on IO.
–	Day 2
4	Find all the real numbers a and b that satisfy the relation $2(a^2+1)(b^2+1) = (a+1)(b+1)(ab+1)$
5	a) Show that there are five integers A, B, C, D, and E such that $2018 = A^5 + B^5 + C^5 + D^5 + E^5$ b) Show that there are no four integers A, B, C and D such that $2018 = A^5 + B^5 + C^5 + D^5$
6	Two players Arnaldo and Betania play alternately, with Arnaldo being the first to play. Initially there are two piles of stones containing x and y stones respectively. In each play, it is possible to perform one of the following operations: 1. Choose two non-empty piles and take one stone from each pile. 2. Choose a pile with an odd amount of stones, take one of their stones and, if possible, split into two piles with the same amount of stones. The player who cannot perform either of operations 1 and 2 loses. Determine who has the winning strategy based on x and y.