

International Zhautykov Olympiad 2020

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- 1 Given natural number n such that, for any natural a, b number $2^a 3^b + 1$ is not divisible by n . Prove that $2^c + 3^d$ is not divisible by n for any natural c and d

- 2 Each of $2k + 1$ distinct 7-element subsets of the 20 element set intersects with exactly k of them. Find the maximum possible value of k .

- 3 Given convex hexagon $ABCDEF$, inscribed in the circle. Prove that $AC * BD * DE * CE * EA * FB \geq 27AB * BC * CD * DE * EF * FA$

- 4 In a scalene triangle ABC I is the incentr and CN is the bisector of angle C . The line CN meets the circumcircle of ABC again at M . The line l is parallel to AB and touches the incircle of ABC . The point R on l is such. That $CI \perp IR$. The circumcircle of MNR meets the line IR again at S . Prpve that $AS = BS$.

- 5 Let Z be the set of all integers. Find all the function $f : Z \rightarrow Z$ such that $f(4x + 3y) = f(3x + y) + f(x + 2y)$
For all integers x, y

- 6 Some squares of a $n \times n$ tabel ($n > 2$) are black, the rest are withe. In every white square we write the number of all the black squares having at least one common vertex with it. Find the maximum possible sum of all these numbers.