

**Regional Competition For Advanced Students 2003**[www.artofproblemsolving.com/community/c1044444](http://www.artofproblemsolving.com/community/c1044444)

by FelixD

- 1 Find the minimum value of the expression  $\frac{a+1}{a(a+2)} + \frac{b+1}{b(b+2)} + \frac{c+1}{c(c+2)}$ , where  $a, b, c$  are positive real numbers with  $a + b + c \leq 3$ .

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- 2 Find all prime numbers  $p$  with  $5^p + 4p^4$  is the square of an integer.

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- 3 Given are two parallel lines  $g$  and  $h$  and a point  $P$ , that lies outside of the corridor bounded by  $g$  and  $h$ . Construct three lines  $g_1, g_2$  and  $g_3$  through the point  $P$ . These lines intersect  $g$  in  $A_1, A_2, A_3$  and  $h$  in  $B_1, B_2, B_3$  respectively. Let  $C_1$  be the intersection of the lines  $A_1B_2$  and  $A_2B_1$ ,  $C_2$  be the intersection of the lines  $A_1B_3$  and  $A_3B_1$  and let  $C_3$  be the intersection of the lines  $A_2B_3$  and  $A_3B_2$ . Show that there exists exactly one line  $n$ , that contains the points  $C_1, C_2, C_3$  and that  $n$  is parallel to  $g$  and  $h$ .

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- 4 For every real number  $b$  determine all real numbers  $x$  satisfying  $x - b = \sum_{k=0}^{\infty} x^k$ .