Regional Competition For Advanced Students 2002

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by parmenides51

1 Find the smallest natural number $x > 0$ so that all following fractions are simplified $\frac{3x+9}{8}, \frac{3x+10}{9}, \frac{3x+11}{10}, ..., 3x$
, i.e. numerators and denominators are relatively prime.

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2 Solve the following system of equations over the real numbers: $2x_1 = x_5^2 - 23$ $4x_2 = x_1^2 + 7$ $6x_3 = x_2^2 + 14$ $8x_4 = x_3^2 + 23$ $10x_5 = x_4^2 + 34$

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3 In the convex $ABCDEF$ (has all interior angles less than 180^o) with the perimeter s the triangles ACE and BDF have perimeters u and v respectively.
a) Show the inequalities $\frac{1}{2} \le \frac{s}{u+v} \le 1$
b) Check whether 1 is replaced by a smaller number or $1/2$ by a larger number can the inequality remains valid for all convex hexagons.

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4 Let $a_0, a_1, ..., a_{2002}$ be real numbers.
a) Show that the smallest of the values $a_k(1 - a_{2002-k})$ $(0 \le k \le 2002)$ the following applies: it is smaller or equal to $1/4$.
b) Does this statement always apply to the smallest of the values $a_k(1-a_{2003-k})$ $(1 \le k \le 2002)$?
c) Show for positive real numbers $a_0, a_1, ..., a_{2002}$:
the smallest of the values $a_k(1 - a_{2003-k})$ $(1 \le k \le 2002)$ is less than or equal to $1/4$.
