

Regional Competition For Advanced Students 2001

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- 1 Let n be an integer. We consider $s(n)$, the sum of the 2001 powers of n with the exponents 0 to 2000. So $s(n) = \sum_{k=0}^{2000} n^k$. What is the unit digit of $s(n)$ in the decimal system?
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- 2 Find all real solutions to the equation

$$(x+1)^{2001} + (x+1)^{2000}(x-2) + (x+1)^{1999}(x-2)^2 + \dots + (x+1)^2(x-2)^{1999} + (x+1)^{2000}(x-2) + (x+1)^{2001} = 0$$

- 3 In a convex pentagon $ABCDE$, the area of the triangles ABC , ABD , ACD and ADE are equal and have the value F . What is the area of the triangle BCE ?
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- 4 Let $A_0 = \{1, 2\}$ and for $n > 0$, A_n results from A_{n-1} by adding the natural numbers to A_{n-1} which can be represented as the sum of two different numbers from A_{n-1} . Let $a_n = |A_n|$ be the number of numbers in A_n . Determine a_n as a function of n .
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