Mathematical Olympiad 2020

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1 A *T-tetromino* is formed by adjoining three unit squares to form a 1×3 rectangle, and adjoining on top of the middle square a fourth unit square.
Determine the least number of unit squares that must be removed from a 202×202 grid so that it can be tiled using T-tetrominoes.

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2 Determine all positive integers k for which there exist positive integers r and s that satisfy the equation

$$(k^2 - 6k + 11)^{r-1} = (2k - 7)^s.$$

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3 Define the sequence $\{a_i\}$ by $a_0 = 1$, $a_1 = 4$, and $a_{n+1} = 5a_n - a_{n-1}$ for all $n \geq 1$. Show that all terms of the sequence are of the form $c^2 + 3d^2$ for some integers c and d.

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4 Let $\triangle ABC$ be an acute triangle with circumcircle Γ and D the foot of the altitude from A. Suppose that $AD = BC$. Point M is the midpoint of DC, and the bisector of $\angle ADC$ meets AC at N. Point P lies on Γ such that lines BP and AC are parallel. Lines DN and AM meet at F, and line PF meets Γ again at Q. Line AC meets the circumcircle of $\triangle PNQ$ again at E. Prove that $\angle DQE = 90^\circ$.
