AoPS Online
Art of Problem Solving

Czech-Polish-Slovak Junior Match 2016

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by parmenides51

–	Individual
1	Let AB be a given segment and M be its midpoint. We consider the set of right-angled triangles ABC with hypotenuses AB. Denote by D the foot of the altitude from C. Let K and L be feet of perpendiculars from D to the legs BC and AC, respectively. Determine the largest possible area of the quadrilateral $MKCL$. Czech Republic
2	Let x and y be real numbers such that $x^2 + y^2 - 1 < xy$. Prove that $x + y - \|x - y\| < 2$. Slovakia
3	Find all integers $n \ge 3$ with the following property: it is possible to assign pairwise different positive integers to the vertices of an n-gonal prism in such a way that vertices with labels a and b are connected by an edge if and only if $a\|b$ or $b\|a$. Poland
4	We are given an acute-angled triangle ABC with $AB < AC < BC$. Points K and L are chosen on segments AC and BC, respectively, so that $AB = CK = CL$. Perpendicular bisectors of segments AK and BL intersect the line AB at points P and Q, respectively. Segments KP and LQ intersect at point M. Prove that $AK + KM = BL + LM$. Poland
5	Determine the smallest integer j such that it is possible to fill the fields of the table 10×10 with numbers from 1 to 100 so that every 10 consecutive numbers lie in some of the $j \times j$ squares of the table. Czech Republic
–	Team
1	Let ABC be a right-angled triangle with hypotenuse AB. Denote by D the foot of the altitude from C. Let Q, R, and P be the midpoints of the segments AD, BD, and CD, respectively. Prove that $\angle APB + \angle QCR = 180^o$. Czech Republic

2 Find the largest integer d divides all three numbers abc, bca and cab with a, b and c being some nonzero and mutually different digits.

Czech Republic

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3 On a plane several straight lines are drawn in such a way that each of them intersects exactly 15 other lines. How many lines are drawn on the plane? Find all possibilities and justify your answer.

Poland

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4 Several tiles congruent to the one shown in the picture below are to be fit inside a 11×11 square table, with each tile covering 6 whole unit squares, no sticking out the square and no overlapping.
(a) Determine the greatest number of tiles which can be placed this way.
(b) Find, with a proof, all unit squares which have to be covered in any tiling with the maximal number of tiles.
`https://cdn.artofproblemsolving.com/attachments/c/d/23d93e9d05eab94925fc54006fe05123f0dba`
`png`
Poland

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5 Let ABC be a triangle with $AB : AC : BC = 5 : 5 : 6$. Denote by M the midpoint of BC and by N the point on the segment BC such that $BN = 5 \cdot CN$. Prove that the circumcenter of triangle ABN is the midpoint of the segment connecting the incenters of triangles ABC and ABM.

Slovakia

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6 Let k be a given positive integer. Find all triples of positive integers a, b, c, such that $a + b + c = 3k + 1$, $ab + bc + ca = 3k^2 + 2k$.

Slovakia
