Finals 1986

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–	Day 1
1	A square of side 1 is covered with m^2 rectangles. Show that there is a rectangle with perimeter at least $\frac{4}{m}$.
2	Find the maximum possible volume of a tetrahedron which has three faces with area 1.
3	p is a prime and m is a non-negative integer $< p-1$. Show that $\sum_{j=1}^{p} j^m$ is divisible by p.
–	Day 2
4	Find all n such that there is a real polynomial $f(x)$ of degree n such that $f(x) \geq f'(x)$ for all real x.
5	There is a chess tournament with $2n$ players ($n > 1$). There is at most one match between each pair of players. If it is not possible to find three players who all play each other, show that there are at most n^2 matches. Conversely, show that if there are at most n^2 matches, then it is possible to arrange them so that we cannot find three players who all play each other.
6	ABC is a triangle. The feet of the perpendiculars from B and C to the angle bisector at A are K, L respectively. N is the midpoint of BC, and AM is an altitude. Show that K, L, N, M are concyclic.