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1 An infinite rectangular stripe of width 3 cm is folded along a line. What is the minimum possible area of the region of overlapping?

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2 Let a real function f defined on the real numbers satisfy the following conditions:
(i) $f(10+x) = f(10-x)$
(ii) $f(20+x) = -f(20-x)$
for all x. Prove that f is odd and periodic.

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3 The positive quadrant of a coordinate plane is divided into unit squares by lattice lines. Is it possible to color the squares in black and white so that:
(i) In every square of side n ($n \in N$) with a vertex at the origin and sides are parallel to the axes, there are more black than white squares;
(ii) Every diagonal parallel to the line $y = x$ intersects only finitely many black squares?

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4 Let $ABCD$ be a tetrahedron. Let a be the length of AB and let S be the area of the projection of the tetrahedron onto a plane perpendicular to AB. Determine the volume of the tetrahedron in terms of a and S.

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5 Let X be the set of natural numbers whose all digits in the decimal representation are different. For $n \in N$, denote by A_n the set of numbers whose digits are a permutation of the digits of n, and d_n be the greatest common divisor of the numbers in A_n. (For example, $A_{1120} = \{112, 121, ..., 2101, 2110\}$, so $d_{1120} = 1$.)
Find the maximum possible value of d_n.

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6 A tourist wants to visit each of the ten cities shown on the picture. The continuous segments on the picture denote railway lines, whereas the dashed segments denote air lines. A railway line costs 150000 lires, and an air line costs 250000 lires. What is the minimum possible price of a desired route?