ITAMO 1995

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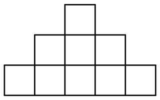
by parmenides51

1 Determine for which values of n it is possible to tile a square of side n with figures of the type shown in the picture



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2 No two of 20 students in a class have the same scores on both written and oral examinations in mathematics. We say that student A is better than B if his two scores are greater than or equal to the corresponding scores of B. The scores are integers between 1 and 10.
(a) Show that there exist three students A, B, C such that A is better than B and B is better than C.
(b) Would the same be true for a class of 19 students?

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3 In a town there are four pubs, A, B, C, D, and any two of them are connected to each other except A and D. A drunkard wanders about the pubs starting with A and, after having a drink, goes to any of the pubs directly connected, with equal probability.
(a) What is the probability that the drunkard is at C at its fifth drink?
(b) Where is the drunkard most likely to be after n drinks ($n > 5$)?

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4 An acute-angled triangle ABC is inscribed in a circle with center O. The bisector of $\angle A$ meets BC at D, and the perpendicular to AO through D meets the segment AC in a point P. Show that $AB = AP$.

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5 Two non-coplanar circles in space are tangent at a point and have the same tangents at this point. Show that both circles lie on some sphere.

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6 Find all pairs of positive integers x, y such that $x^2 + 615 = 2^y$