Dutch Mathematical Olympiad 1995

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by moldovan

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1   A kangaroo jumps from lattice poin to lattice point in the coordinate plane. It can make only two kinds of jumps: (A) 1 to right and 3 up, and (B) 2 to the left and 4 down. (a) The start position of the kangaroo is $(0,0)$. Show that it can jump to the point $(19,95)$ and determine the number of jumps needed. (b) Show that if the start position is $(1,0)$, then it cannot reach $(19,95)$. (c) If the start position is $(0,0)$, find all points (m,n) with $m,n \geq 0$ which the kangaroo can reach.

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2   For any point P on a segment AB, isosceles and right-angled triangles AQP and PRB are constructed on the same side of AB, with AP and PB as the bases. Determine the locus of the midpoint M of QR when P describes the segment AB.

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3   Let 101 marbles be numbered from 1 to 101. The marbles are divided over two baskets A and B. The marble numbered 40 is in basket A. When this marble is removed from basket A and put in B, the averages of the numbers A and B both increase by $\frac{1}{4}$. How many marbles were there originally in basket A?

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4   A number of spheres with radius 1 are being placed in the form of a square pyramid. First, there is a layer in the form of a square with n^2 spheres. On top of that layer comes the next layer with $(n-1)^2$ spheres, and so on. The top layer consists of only one sphere. Compute the height of the pyramid.

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5   An array $(a_1, a_2, ..., a_{13})$ of 13 integers is called $tame$ if for each $1 \leq i \leq 13$ the following condition holds: If a_i is left out, the remaining twelve integers can be divided into two groups with the same sum of elements. A tame array is called $turbo\ tame$ if the remaining twelve numbers can always be divided in two groups of six numbers having the same sum. (a) Give an example of a tame array of 13 integers (not all equal). (b) Prove that in a tame array all numbers are of the same parity. (c) Prove that in a turbo tame array all numbers are equal.