Dutch Mathematical Olympiad 1993

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1 Show that any subset of $V = \{1, 2, ..., 24, 25\}$ with 17 or more elements contains at least two distinct numbers the product of which is a perfect square.

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2 In a triangle ABC with $\angle A = 90^\circ$, D is the midpoint of BC, F that of AB, E that of AF and G that of FB. Segment AD intersects CE, CF and CG in P, Q and R, respectively. Determine the ratio: $\frac{PQ}{QR}$.

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3 A sequence of numbers is defined by $u_1 = a, u_2 = b$ and $u_{n+1} = \frac{u_n + u_{n-1}}{2}$ for $n \geq 2$. Prove that $\lim_{n \to \infty} u_n$ exists and express its value in terms of a and b.

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4 Let C be a circle with center M in a plane V, and P be a point not on the circle C. (a) If P is fixed, prove that $AP^2 + BP^2$ is a constant for every diameter AB of the circle C. (b) Let AB be a fixed diameter of C and P a point on a fixed sphere S not intersecting V. Determine the points P on S that minimize $AP^2 + BP^2$.

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5 Eleven distinct points $P_1, P_2, ..., P_{11}$ are given on a line so that $P_iP_j \leq 1$ for every i, j. Prove that the sum of all distances $P_iP_j, 1 \leq i < j \leq 11$, is smaller than 30.
