Dutch Mathematical Olympiad 1983

www.artofproblemsolving.com/community/c1060359

by moldovan

1	A triangle ABC can be divided into two isosceles triangles by a line through A. Given that one of the angles of the triangles is 30°, find all possible values of the other two angles.
2	Prove that if n is an odd positive integer, then the last two digits of $2^{2n}(2^{2n+1}-1)$ in base 10 are 28.
3	Suppose that a, b, c, p are real numbers with a, b, c not all equal, such that: $a+\frac{1}{b} = b+\frac{1}{c} = c+\frac{1}{a} = p$. Determine all possible values of p and prove that $abc+p=0$.
4	Within an equilateral triangle of side 15 are 111 points. Prove that it is always possible to cover three of these points by a round coin of diameter $\sqrt{3}$, part of which may lie outside the triangle.