Federal Competition For Advanced Students, Part 2 1990

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–	Day 1
1	Determine the number of integers n with $1 \le n \le N = 1990^{1990}$ such that $n^2 - 1$ and N are coprime.
2	Show that for all integers $n \ge 2$, $\sqrt{2\sqrt[3]{3\sqrt[4]{4...\sqrt[n]{n}}}} < 2$
3	In a convex quadrilateral $ABCD$, let E be the intersection point of the diagonals, and let F_1, F_2, and F be the areas of ABE, CDE, and $ABCD$, respectively. Prove that: $\sqrt{F_1} + \sqrt{F_2} \le \sqrt{F}$.
–	Day 2
4	For each nonzero integer n find all functions $f : \mathbb{R} - \{-3, 0\} \to \mathbb{R}$ satisfying: $f(x+3) + f\left(-\frac{9}{x}\right) = \frac{(1-n)(x^2+3x-9)}{9n(x+3)} + \frac{2}{n}$ for all $x \ne 0, -3$. Furthermore, for each fixed n find all integers x for which $f(x)$ is an integer.
5	Determine all rational numbers r such that all solutions of the equation: $rx^2 + (r+1)x + (r-1) = 0$ are integers.
6	A convex pentagon $ABCDE$ is inscribed in a circle. The distances of A from the lines BC, CD, DE are a, b, c, respectively. Compute the distance of A from the line BE.