Federal Competition For Advanced Students, Part 2 1989

www.artofproblemsolving.com/community/c1060503

by parmenides51, moldovan

–	Day 1
1	Consider the set S_n of all the 2^n numbers of the type $2\pm \sqrt{2 \pm \sqrt {2 \pm ...}}$, where number 2 appears $n+1$ times. (a) Show that all members of S_n are real. (b) Find the product P_n of the elements of S_n.
2	Find all triples (a,b,c) of integers with $abc=1989$ and $a+b-c=89$.
3	Show that it is possible to situate eight parallel planes at equal distances such that each plane contains precisely one vertex of a given cube. How many such configurations of planes are there?
–	Day 2
4	We are given a circle k and nonparallel tangents t_1,t_2 at points P_1,P_2 on k, respectively. Lines t_1 and t_2 meet at A_0. For a point A_3 on the smaller arc $P_1 P_2,$ the tangent t_3 to k at P_3 meets t_1 at A_1 and t_2 at A_2. How must P_3 be chosen so that the triangle $A_0 A_1 A_2$ has maximum area?
5	Find all real solutions of the system: $x^2+2yz=x$, $y^2+2zx=y$, $z^2+2xy=z$.
6	Determine all functions $f : \mathbb{N}_0 \to \mathbb{N}_0$ such that $f(f(n))+f(n)=2n+6$ for all $n \in \mathbb{N}_0$.